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Does public indebtedness constrain or can it favor economic growth? A simple analytical modeling

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Abstract

This paper aims at shedding an analytical light on the consequences of the public indebtedness level on economic growth. We show that increasing the current public debt can sustain short run economic activity, and mainly net exports and public investment expenditure. On the labor market, in the short run, the public debt can also increase the capital stock, the real wage, and more moderately labor demand and supply. However, our modelling also underlines that there are many obstacles to the positive effect of a higher public debt level on long term economic growth. Indeed, if the elasticity of the nominal interest rate to the increase of the public debt becomes high (worry of the financial markets), increasing the public indebtedness level can damage current economic activity. Besides, a previous increasing trend of the public debt can be damaging, and the trajectory of the public debt should also be taken into account.

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1 Introduction

The link between fiscal policy, public debt and economic growth is a major concern in the debate about appropriate and optimal economic policies. However, theoretical or econometric studies don't succeed to underline a clear-cut relation between the size of the government (public expenditure, fiscal burden or even public indebtedness level) and economic growth; empirical analysis don't shed light on an obvious relation. So, are the fiscal burden (a high level of various taxation rates) and public expenditure growth enhancing (positive fiscal multiplier in Keynesian models), or can a fiscal consolidation be expansionary? Is the public debt level harmful to economic growth, or can it be growth enhancing? The question has been largely studied in the economic literature, without having a clear-cut answer.

In the context of the Ricardian equivalence, the public debt has no consequence on the long-term economic growth rate. However, if this equivalence does not hold, the public debt can have important implications on interest rates and on economic growth. A higher public debt would have to be repaid in the future, with higher distortionary taxes on various fiscal instruments, and its consequences are then non-negligible. That's why this public debt is often monitored and constrained, in particular by the Fiscal Compact in the framework of the European Economic and Monetary Union (EMU). For example, Huffman (2013) introduced the existence of a 'debt trigger' in his paper: beyond a given level of the public debt to GDP ratio, taxation rates couldn't avoid to be increased, in order to

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prevent the public debt to exceed a level considered as maximal. Indeed, excessive public debt levels should be avoided, as they would be associated with lower long term GDP growth rates.

In the framework of the EMU, the Stability and Growth Pact and afterwards the Fiscal Compact prevent the budgetary deficit of the member countries to exceed 3 percent of GDP, and they constrain the public debt to be reduced at a satisfactory pace if it exceeds a threshold of 60% of GDP. Indeed, the European authorities believed that an excessively high public debt level in a member country and the weight of the interest rates on this public debt should necessarily imply a weaker level of public expenditure, and /or higher taxes to collect more fiscal resources, with negative externalities on the partner countries of the monetary union. Besides, some economic studies confirm the link between a high public debt level and a lower long-term economic growth. However, what are the opposite economic arguments justifying or on the contrary condemning the increase of the public indebtedness level?

The level of indebtedness, whether private (households, firms) or public (government), is a serious concern for the global well-being and for economic growth. However, even if the relationship between debt and economic growth has been largely studied, the correlation between both variables is not clear cut. Indeed, a moderate indebtedness level can be beneficial and welfare improving, by providing immediate financial resources for a necessary investment useful to future economic growth. Higher taxation and budgetary deficit, and a higher public debt level can contribute to finance productive public expenditure, like education or public infrastructure [see Myles (2008) for this role in endogenous growth models]. The long term equilibrium growth rate and optimal level of public capital stock accumulation are then reached faster. However, even if the public debt is beneficial for current generations, it is generally harmful for future generations. Indeed, an excessive public indebtedness level can make the weight of its reimbursement too heavy, divert too many financial resources from current economic growth, and increase the risk of real volatility and of an economic crisis [see Cecchetti *et al.* (2011)]. High taxation rates are harmful if they are strongly distortionary, and if they imply a decrease of the tax base [see Aizenman *et al.* (2007)]. High taxation rates reduce the returns of any investment, which means less accumulation of capital, less innovation, and less economic growth [Myles (2008)]. Besides, a high public debt level increases long term interest rates, and then implies a risk of crowding-out private investment. That's why even if increasing the public debt is beneficial to boost economic growth in the short run, it often appears harmful to the capital stock and to long term economic growth.

The aim of the current paper is to give an analytical basis to some of these intuitions regarding the consequences of an increase of the public debt on economic growth. Indeed, there are many econometric papers studying the effects of the public debt on economic growth, trying also to find debt thresholds, below which the public debt could be beneficial before becoming harmful (see the economic literature in the following section 2). However, analytical papers and macro-economic models studying this link are much more unusual, and the goal of the current paper is to fill this gap. Indeed, an analytical modelling can contribute to show why and according to which parameters increasing the public debt level can be beneficial to current economic growth, even if the growth enhancing effect is not the same for all factors of global demand. Furthermore, our analytical framework also shows the limits of this growth enhancing effect. First, an excessively high public indebtedness level can increase the nominal interest rate if financial markets fear that this debt becomes unsustainable. Second, we also show that it is important to consider the full trajectory of the public debt, as a past increase of the public debt during the previous period appears harmful to current economic growth.

The rest of the paper is organized as follows. The second section recalls the results of the economic literature regarding the links between fiscal policy, public debt level and economic growth. The third section describes our New-Keynesian model: the economic agents, the fiscal policy and the determinants of the public debt level, as well as the global equilibrium of the model. The fourth section describes the potential consequences of the public debt level for economic growth: the potential growth-enhancing effect of the current public indebtedness, on the various factors of global demand and on the labor market. However, the fifth section underlines the limits of this growth enhancing effect: if the nominal interest rate increases, or if the past growth rate of the public debt increases the weight of the global indebtedness. Finally, the sixth section concludes the paper.

2 The economic literature

The effect of taxation and of public debt on economic growth is not always negative in the economic literature; it is mostly ambiguous [Myles (2008)]. As mentioned by Cottarelli and Jaramillo (2013), first, the public debt influences economic growth because if the fiscal sustainability of an economic policy is put into question by the financial markets, in absence of fiscal adjustment, a financial crisis can occur. However, if fiscal consolidation implies a lower economic growth in the short term, it could also worry and imply higher financing costs on financial markets, even if it improves fiscal sustainability, long term and potential economic growth. For example, Greiner (2014) analyzes how public debt affects the allocation of resources in a basic endogenous growth model where growth results from positive externalities of physical capital. In his model, the primary surplus relative to GDP is a positive linear function of the debt to GDP ratio in order to guarantee the intertemporal sustainability of public debt, and it also depends on cumulated past levels of public debt (history) with exponentially declining weights put on debt further back in time. The author then shows that the higher the debt ratio, the lower the balanced growth rate, so that an economy with a balanced government budget always experiences a higher long-run growth rate than an economy with permanent public deficits.

Teles and Mussolini (2014) present a theoretical model of endogenous growth, which demonstrates that the level of the public debt-to-GDP ratio should negatively impact the effect of productive government expenditure on long-term growth. In a model with overlapping generations and endogenous growth, this occurs because government indebtedness, even devoted to productive public expenditure, extracts part of the savings of the young to pay interest on the debts of the older generation. This exchange between generations reduces the global saving rate of the economy. Furthermore, in the framework of a neoclassical endogenous growth model, Le Van *et al.* (2019) analytically study the impact of various financing of public investment (domestic public debt, external public debt or taxes on returns to private assets) on economic growth. In particular, for the existence of a positive balanced-growth path, the economy should have a sufficiently large productivity and a sufficiently low tax on returns to assets, both characteristics which can be impacted by the nature and by the level of the public indebtedness.

In the same framework of a Barro-type endogenous growth model, Cheron *et al.* (2019) show that the relation between the public debt to GDP ratio and economic growth strongly depends on expectations from economic agents; self-fulfilling expectations may then contribute to explain macroeconomic fluctuations. Indeed, when debt is increasing, the balanced-growth path may be indeterminate; large fluctuations associated to self-fulfilling believes may occur and be associated with welfare losses if there is a coordination on the low steady-state. So, the economic literature has underlined that the influence of the public debt level on economic growth is multi-factorial, and that many factors could complexify the nature of this relation. Caner *et al.* (2019) also underline the importance to study jointly the interaction between private and public debt in influencing economic growth. Indeed, a higher private debt level risks depressing demand and output, and therefore to necessitate also a higher public indebtedness level in the context of a contra-cyclical expansionary budgetary policy. According to the authors, at low levels of debt, this interaction stimulates economic growth, but above a threshold, private-public debt interaction decreases growth. Using data from 29 OECD countries for the 1995-2014 period, they find that the interaction between the public and private debt variables and economic growth is negative and significant when it reaches the level of 137% (endogenous threshold).

Besides, Krogstrup (2002) considers that the public debt is exogenous and has reached its steady state level, and then, he analyses the consequences of asymmetric public debt levels on the fiscal policy of various European countries. Then, cross country differences in public debts are found to imply asymmetries in taxes and primary expenditures across EU countries, with high debt countries having lower expenditures and higher taxes than low debt countries. Furthermore, capital mobility would increase these asymmetries, and trigger cross-country asymmetries in the tax mix. Indeed, the empirical test for a panel of EU countries between 1970 and 1999 shows that high debt countries have smaller public sectors (less public expenditure) and higher taxes than low debt countries. Moreover, regarding the tax mix, tax competition implies that the tendency to increase taxes in high debt countries relies mainly on the less mobile labor and consumption factors. Furthermore, Sosvilla-Rivero and Gómez-Puig (2019) employ the Autoregressive Distributed Lag bounds testing approach using annual data from eleven both central and peripheral countries of the European Economic and Monetary Union (EMU) for the 1961-2015 period. Their results suggest that the impact of public debt on economic growth not only changes across EMU countries, but also over time (with macroeconomic, financial or institutional factors), allowing for different endogenously defined (data based) regimes. More precisely, the negative impact of an excessive public debt level on economic growth mainly happens after exceeding a breakpoint of economic and fiscal distress, which can differ among countries.

Therefore, econometric studies usually conclude that the effect of the public debt on economic growth is mostly ambiguous. Empirically, Panizza and Presbitero (2014) show no causal effect between the public debt level and economic growth in advanced economies. Besides, they argue that it is the lower growth that can result in higher debt to GDP ratios, and not the contrary. Kourtellos *et al.* (2013) use a structural threshold regression, for 82 countries, between 1980 and 2009. They find strong evidence for threshold effects based on democracy, as a proxy for institutional quality: higher public debt results in lower growth only for countries in the Low-Democracy regime. Pescatori *et al.* (2014) find no evidence of any particular debt threshold above which medium-term growth prospects are dramatically compromised. Nevertheless, a higher debt seems to be associated with a higher degree of output volatility. Furthermore, they find that the debt trajectory can be as important as the debt level in influencing the sustainability of the public debt level and in understanding future growth prospects, since countries with high but declining debt appear to grow equally as fast as countries with lower debt. Analyzing over two centuries of data for the United States, Great Britain, Sweden and Japan, Eberhardt (2013) finds very limited evidence for nonlinear long-run relationships in these countries, and further cannot support the notion that their equilibrium debt growth relationship is identical. Thresholds seem to differ across countries and according to their global economic framework.

The study of the IMF (2008), using data for the period 1970-2007, demonstrates the following, both for advanced and emerging economies: economies that implement fiscal stimulus and have high public debts going into a downturn typically experience lower growth rates before and after the downturn year and less of a pickup in growth in the year following the fiscal stimulus, whereas high-debt economies that implement fiscal tightening experience stronger gains in growth. Therefore, this study shows that the effect of a fiscal stimulus is consistently large and persistently negative for highly indebted countries. Indeed, increases in interest rate risk premiums as a result of debt concerns can render fiscal multipliers negative, making a discretionary fiscal stimulus harmful. In the same way, Antonakakis (2014) studies 12 Euro area (EA) countries over the period 1970-2013, He finds that non-sustainable debt-ratios above and below the 60% threshold have a detrimental effect on short-run economic growth, while sustainable debt-ratios below the 90% threshold exert a positive influence on short-run economic growth. In the long-run, both non-sustainable and sustainable debt-ratios above the 90% threshold, as well as non-sustainable debt-ratios below the 60% compromise economic growth. Baglan and Yoldas (2013) explore the empirical relationship between government debt and future macroeconomic activity using data on twenty advanced economies throughout the postwar era. Despite a large degree of uncertainly in these regressions, for countries with relatively low average debt ratios (below 20% of GDP), they find a negative threshold effect as their debt ratios increase toward moderate levels. For countries with chronically high debt ratios, GDP growth slows quite linearly as relative government debt increases, without any significant threshold effect.

Baum *et al.* (2013) focus on 12 Euro Area countries for the period 1990-2010, with a dynamic threshold panel methodology. They show that the short-run impact of debt on GDP growth is positive and highly statistically significant, but it would be non-linear: it decreases to around zero and loses significance for public debt-to-GDP ratios beyond 67%. For high debt-to-GDP ratios (above 95%), any additional debt has a negative impact on economic activity. In the same way, Cecchetti *et al.* (2011) study a dataset of 18 OECD countries from 1980 to 2010, and they find that, beyond a given threshold around 85% of GDP, government debt is a drag on growth. It is valid for other types of debts: when corporate debt goes beyond 90% and when the debt of households goes beyond a more imprecise value around 85% of GDP, it becomes harmful to economic growth. Checherita and Rother (2012) investigate the average impact of government debt on per-capita GDP growth in twelve Euro area countries over the period 1970-2011. They also find a non-linear impact of debt on growth with a turning point—beyond which the government debt-to-GDP ratio has a deleterious impact on long-term growth—at about 90-100% of GDP. Confidence intervals for this debt turning point suggest that the negative growth effect of high debt may start already from levels around 70-80% of GDP, which calls for even more prudent indebtedness policies. So, the main result of most econometric studies is as follows: if a moderate level of public indebtedness could be beneficial in the short run, there would certainly be a threshold that shouldn't be reached, and beyond which the public debt level would be harmful to long run economic growth.

In this framework; one of the most influential paper is the one by Reinhart and Rogoff (2010). They study 44 countries for a period of about two hundred years. They find that the relationship between government debt and real GDP growth is weak for debt/GDP ratios below a threshold of 90% of GDP. However, above 90%, median growth rates fall by one percent, and average growth falls considerably more (-0.1%). They find that the threshold for public debt would be similar in advanced and emerging economies; the main difference is then that inflation rates are much higher for high debt levels in emerging economies. Nevertheless, the striking results of this paper have been afterwards put into question. Indeed, Herndon *et al.* (2014) find that for the same 20 advanced countries for 1946-2009, after correcting for coding and weighting of the data errors, the average real GDP growth rate for countries carrying a public-debt-to-GDP ratio of over 90% is 2.2%, not so dramatically different from when debt/GDP ratios are lower. Besides, Minea and Parent (2012) use up-to-date econometric techniques (Panel Smooth Threshold Regression), and they reveal an endogenously-estimated threshold around a debt-to-GDP ratio of 115%. Below this threshold, a debt increase damages growth; however, this negative effect is declining as public debt is increasing. Furthermore, above this threshold, the link between public debt and economic growth even changes sign, and a higher public debt could then increase growth. Egert (2015a) also shows that the threshold of 90% of GDP is not very robust for the period 1946-2009. According to him, a high public debt implies poor economic performances at much weaker levels (around 20% or 30% of GDP for the debt of the central government) if the threshold is endogenously defined, without any further increase beyond 90% of GDP. Besides, there would be large cross countries differences, and variations within countries across time; results would be highly sensitive to modelling choices (choice of periods and of countries, data frequency). Using nonlinear threshold models, Egert (2015b) finds that GDP growth would decrease gradually when the public debt rises from 30% to above 90%, without abrupt threshold. These results, based on bivariate regressions on secular time series (1790-2009), are confirmed on a shorter dataset (1960-2010) using a multivariate growth framework.

Woo and Kumar (2015) also explore the impact of high public debt on long-run economic growth, with a panel of advanced and emerging economies in the period 1970-2007. The empirical results suggest an inverse relationship between initial debt and subsequent growth, controlling for other determinants of growth: on average, a 10 percentage point increase in the initial debt-to-GDP ratio is associated with a subsequent slowdown in annual real per capita GDP growth of around 0.25 percentage points per year for the next 5 years, with the impact being somewhat smaller in advanced economies. Besides, there would be some evidence of nonlinearity, with higher levels of initial debt having a proportionately larger negative effect on subsequent growth. Analysis of the components of growth suggests that the adverse effect largely reflects a slowdown in labor productivity growth mainly due to reduced investment and slower growth of capital stock. In the same way, for 101 developing and developed economies from 1980 to 2008, Caner *et al.* (2010) find that the negative effect on long term economic growth would begin after a threshold for the long term public debt around 77% of GDP, and as soon as 64% of GDP in emerging markets. However, the initial GDP per capita ratio should also be taken into account, especially in low debt countries.

Other papers mainly underline the fact that accepting a higher public debt level could be positive in the short term but detrimental in the long term. For example, Aizenman *et al.* (2007) use a non-stochastic endogenous growth model. They show that public debt should not finance current public consumption (flow spending). Regarding the investment in the stock of public infrastructure, public sector borrowing to finance the accumulation of public capital may allow the economy to reach a long-run optimal and sustainable growth path faster. However, a high level of public debt reduces this equilibrium growth rate. So, with either a binding exogenous debt limit or a solvency constraint, a more patient country will have a higher steady-state growth rate, but a longer transition with a lower short term growth rate. In the same way, Cottarelli, and Jaramillo (2013) underline the necessity to reduce high public debt to GDP ratios, because they penalize potential and long-term growth (cf. the situations of Japan or Italy). However, fiscal adjustment is likely to hurt growth in the short run, delaying improvement in fiscal indicators. Therefore, they favor a gradual adjustment of the public debt at a steady pace, rather than a front-loaded adjustment, provided market pressures do not require a faster consolidation.

The high current public debt level in the European countries is mainly a legacy of the financial crisis of 2008. Therefore, this high public indebtedness level has revived the debate about its consequences on potential economic growth. Reinhart *et al.* (2015) mention that this deterioration in the fiscal framework is mainly due to discretionary budgetary policies of the governments to avoid the recession, but that it strongly impairs future prospects of economic growth. However, the authors also mention the political reluctance of governments to have excessively high public indebtedness levels for a long period. So, they explore the inverse causality, and the possibility that an orthodox way to return to more moderate public debt levels in the current period would be to attain growth rates higher than interest rates. More heterodox policies to reduce public debt levels would be debt restructuring (mainly regarding external debt), eroding debt in real terms through unexpected inflation, or keeping its real cost low through 'financial repression'.

Another harmful effect of a high public debt level is underlined by Janeba and Todtenhaupt (2016). Indeed, they develop a simple model of fiscal competition with government borrowing. Then, if a default on government debt is no option, initial debt levels play no role in fiscal competition. To the contrary, a government can be constrained in its borrowing, due to a possible default, or simply because of common rules of fiscal discipline (like the Fiscal Compact in the European Union). Therefore, in these conditions, a government responds optimally by lowering spending on durable public infrastructure, which in turn induces more aggressive tax setting. So, their model may help to explain the observation that highly indebted countries in Europe have decreased corporate tax rates over-proportionally, and have infrastructure which continue to deteriorate. Besides, a rise in exogenous firm mobility reinforces the link between legacy debt and fiscal competition. The cut in capital taxation rates risks to be all the more aggressive as the high inherited public debt level is due to high previous public consumption expenditure, with an insufficient level of public investment expenditure in infrastructure.

However, Huffman (2013) shows that higher levels of government debt do not necessarily reduce growth rates contemporaneously; a higher public debt may even increase the short-term growth rate. Indeed, as more consumption is shifted into the future when taxation rates will be higher, this increases current sparing and investment. Therefore, the current public debt level can have ambiguous implications on the current growth rate, depending on the nature of the budgetary constraint (effect of a higher future distortionary tax on consumption and investment decisions and on other parameters of budgetary policy) and on the consumer's intertemporal elasticity of substitution of consumption. Nevertheless, in the medium term, approaching the debt trigger, higher taxes will reduce the growth rate. Indeed, according to Huffman (2013), beyond a high level of 100 percent of GDP (the 'debt trigger'), the public debt would automatically imply increases of future taxation rates in order to reduce the public indebtedness and to satisfy the budgetary constraint.

In line with the previous studies, the aim of the current paper is to study analytically the implications of a variation of the public indebtedness level in the short run, in comparison with the last period's value (influence of the debt trajectory), but also according to its absolute value and to its weight for the public finances.

3 The model

We consider a small New-Keynesian model, with a representative household, a representative firm and a government in a given country (i). We study and model the situation of this country, which is member of a monetary union. This implies that the country (i) is constrained to share the common nominal interest rate defined by the common central bank at the level of the whole monetary union. Therefore, we suppose that this common interest rate is exogenously fixed for the country (i) (we don't detail its determination in the framework of our modeling), because we make the hypothesis that the country (i) is too small to influence the common monetary policy by its specific economic conditions. Studying the situation of a member country of a monetary union also implies constraints on the fixation of the capital (mobile factor) taxation rate; indeed, financial markets are assumed to be complete both at the national and international level (risks are fully shared among households). On the contrary, the government of the country (i) can define autonomously its fiscal policy (public expenditure, tax revenues and public debt), in order to maximize the utility of a representative consumer; it may levy taxes on labor, consumption and capital. Income distribution issues are ignored by assuming either that each region's residents are identical or that their aggregate welfare can be depicted by the preferences of a 'representative consumer'.

Furthermore, in the current paper, we suppose that capital is taxed according to the source base principle. Indeed, taxing residents on their world-wide capital income equally, according to the residence principle, is very difficult empirically: administrative and tax compliance problems involved in taxing foreign source income, imperfect exchange of information among the tax authorities, persistence of bank secrecy laws, etc. Besides, in the area of corporate income taxation, many residence countries explicitly exempt foreign-source income from domestic tax if the foreign income originates from a tax treaty partner country. Most other countries only tax the foreign-source income of their 'resident' multinationals to the extent that this income is repatriated to the parent company, and only in so far as the domestic tax liability exceeds the source tax which has already been paid to the foreign country. So, we make the hypothesis that at least regarding (corporate) taxes on capital, residents of a country are not taxed on their income from foreign sources and that foreigners are taxed equally as residents on income from domestic sources.

3.1 The representative consumer

Aggregate demand for the country (i) results from the log-linearization of the Euler equation, which describes the representative household's expenditure decisions. The representative consumer in the country (i) provides labor and it consumes goods. In a given period (T), it maximizes an inter-temporal utility function:

$$max \sum_{t=T}^{\infty} \beta^{t-T} E_T[U_{i,t}] \quad (1)$$

Where: $E_t()$ is the rational expectation operator conditional on information available at date (t), and (β) is the time discount factor (preference for the future). Interest rates, taxation rates, prices and wages are then taken as given by the representative consumer.

We suppose that the utility function of a representative household is as follows:

$$U_{i,t} = \alpha_c \ln(C_{i,t}) + \alpha_g \ln(G_{i,t}) - \alpha_l \frac{1}{(1+\varphi)} L_{i,t}^{s(1+\varphi)} \quad (2)$$

With: ($C_{i,t}$): real consumption of private goods; ($G_{i,t}$): real public expenditure (consumption of public goods); ($L_{i,t}^s$): Labor supply.

The indices ($0<\alpha_c<1$) ($0<\alpha_g<1$) and ($0<\alpha_l<1$) are the respective weights given by the representative consumer to consumption of private goods, public goods and leisure.

Utility is an increasing and concave function of ($C_{i,t}$), an index of the household's private consumption of all goods that are supplied, and of public goods and services provided in the home country ($G_{i,t}$). Utility is also a decreasing and convex function of labor supply ($L_{i,t}^s$), where ($\varphi \geq 0$) is the inverse of the Frisch labor supply elasticity.

This maximization is subject to the life time and inter-temporal budgetary constraint. If we suppose complete financial markets, the budgetary constraint for each period (t) of the representative consumer in the country (i) is as follows:

$$\begin{aligned}(1+t_{i,t}^c)P_{i,t}C_{i,t} + P_{i,t}INV_{ii,t} + P_{u,t}INV_{iu,t} + B_{i,t} &= (1-t_{i,t}^k)(R_t - \delta P_{i,t})K_{ii,t}^s \\ &+ (1-t_{u,t}^k)(R_t - \delta P_{u,t})K_{iu,t}^s + (1-t_{i,t}^l)W_{i,t}L_{i,t}^s + (1+R_{t-1})B_{i,t-1} \quad (3)\end{aligned}$$

With, in period (t), where (j) is either the country (i) or the rest of the monetary union (u): ($INV_{ij,t}$): real investment of households from country (i) in new physical capital in country (j); ($K_{ij,t}^s$): physical capital belonging to households in country (i) invested in country (j); ($P_{i,t}$): consumer prices; ($W_{i,t}$):

nominal wage rate; (R_t): nominal interest rate common to all countries in the monetary union; $(B_{i,t})$: nominal value of government' bonds and public debt at the end of period (t); (δ): depreciation rate of physical capital; $(t^l_{i,t})$: labor taxation rate; $(t^c_{i,t})$: consumption taxation rate; $(t^k_{i,t})$: capital taxation rate.

Indeed, the representative consumer of the country (i) may consume his non-human wealth immediately, or he may invest it on the capital market and consume it at the end of the period. So, regarding his expenditure, he consumes private goods, and he invests in capital or he purchases government' bonds. Regarding his resources, he receives labor (wage) and capital (interest rate) revenues. Indeed, we suppose that capital is rented by households to firms, for which they receive a rental rate. The representative consumer also receives gains from government bonds holding from the previous period. For simplicity, we suppose that these financial assets are only riskless one-period government bonds, and that the public debt of the country (i) is fully owned by domestic consumers. Besides, capital is not fully taxed: physical capital depreciation is exempted from taxation.

In this context, the maximization of equation (1) using (2) under the constraint (3) implies the following first order Euler condition, regarding timing of consumption expenditure decisions and inter-temporal substitution, (∀k):

$$C_{i,t} = \frac{E_t[(1+t^c_{i,t+k})P_{i,t+k}]}{\beta^k E_t[(1+R_{t+k-1})\dots(1+R_t)](1+t^c_{i,t})P_{i,t}} E_t(C_{i,t+k}) \quad (4)$$

So, in logarithms and in variation from its last period's value, with $[\pi_{i,t} = \frac{P_{i,t}-P_{i,t-1}}{P_{i,t-1}}]$: inflation rate for consumption prices, equation (4) implies:

$$c_{i,t} = E_t(c_{i,t+1}) - [R_t - R_{t-1} - E_T(\pi_{i,t+1}) + \pi_{i,t}] + \ln\left[\frac{(1+t^c_{i,t-1})E_t(1+t^c_{i,t+1})}{(1+t^c_{i,t})^2}\right] \quad (5)$$

For each variable X, its variation rate is: $x_{i,t} = lnX_{i,t} - lnX_{i,t-1} = \frac{X_{i,t}-X_{i,t-1}}{X_{i,t-1}}$.

So, private consumption increases with expected future consumption, and it decreases with the growth of the real interest rate, and with the temporary increase of the consumption taxation rate in comparison with its past and expected future level.

Besides, for the representative consumer in the country (i), we obtain the following optimal substitution between private consumption, public consumption and working time[2]:

$$\frac{\partial U_{i,t}}{\partial C_{i,t}} = -\frac{(1+t^c_{i,t})P_{i,t}}{W_{i,t}(1-t^l_{i,t})}\frac{\partial U_{i,t}}{\partial L^S_{i,t}} = \frac{\partial U_{i,t}}{\partial G_{i,t}} \quad (6)$$

So, regarding labor supply, according to equations (2) and (6), in variation from its last period's value, we obtain[3]:

$$l^S_{i,t} = \frac{1}{\varphi}(w_{i,t} - \pi_{i,t}) + \frac{1}{\varphi} ln\left[\frac{(1-t^l_{i,t})(1+t^c_{i,t-1})}{(1+t^c_{i,t})(1-t^l_{i,t-1})}\right] - \frac{1}{\varphi}c_{i,t} \quad (7)$$

Labor supply increases with the real wage, but it decreases with the temporary increase of labor and consumption taxation rates ($t^l_{i,t}$ and $t^c_{i,t}$) in comparison with their values in the previous period, reducing the purchasing power for households of a given labor revenue, and it also decreases with the disutility of working time (φ).

Furthermore, regarding variations of private and public consumption, with equations (2) and (6), in variation from their last period's values, we obtain[4]:

$$g_{i,t} = c_{i,t} \quad (8)$$

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[2] Here, we use equations (3) and (20), which imply:

$$P_{i,t}(C_{i,t}+G_{i,t}) = (R_t - \delta P_{i,t})K_{ii,t} - P_{i,t}INV_{ii,t} + (1-t^k_{u,t})(R_t - \delta P_{u,t})K_{iu,t} - P_{u,t}INV_{iu,t} + t^k_{i,t}(R_t - \delta P_{i,t})K_{ui,t} + W_{i,t}L_{i,t}..$$

[3] $\frac{\partial U_{i,t}}{\partial L^S_{i,t}} = -\alpha_l (L^S_{i,t})^{\varphi} = -\frac{W_{i,t}(1-t^l_{i,t})}{(1+t^c_{i,t})P_{i,t}}\frac{\partial U_{i,t}}{\partial C_{i,t}} = -\frac{W_{i,t}(1-t^l_{i,t})}{(1+t^c_{i,t})P_{i,t}}\frac{\alpha_c}{(C_{i,t})}$.

[4] $\frac{\partial U_{i,t}}{\partial G_{i,t}} = \frac{\alpha_g}{(G_{i,t})} = \frac{\partial U_{i,t}}{\partial C_{i,t}} = \frac{\alpha_c}{(C_{i,t})}$.

Therefore, private and public consumption should increase at the same pace, in order to maximize the utility of the representative consumer.

Finally, we suppose that the capital stock varies according to the following equation:

$$E_t(K_{ij,t+1}) = (1-\delta)K_{ij,t} + INV_{ij,t} \quad (9)$$

So, in variation from its last period's value, the capital stock adjusts as follows:

$$E_t(k_{ij,t+1}) = (1-\delta)k_{ij,t} + \left(\frac{INV_{ij}}{K_{ij}}\right)inv_{ij,t} = (1-\delta)k_{ij,t} + \delta inv_{ij,t} \quad (10)$$

3.2 The representative firm

We suppose a continuum of firms in the country (i). The representative firm produces a differentiated good in a monopolistic competition setting, with the help of two production factors: capital (from domestic or foreign source) and labor, which respective shares in the production function are $(0<\nu<1)$ and $(0<1-\nu<1)$. We assume that marginal products are positive and diminishing, and that all factors are complement in the production function. Besides, public expenditure is also a factor raising public input; production increases with public goods and services supplied by the government. However, we suppose that investment public expenditure ($G_{inv.i,t}$) is more productive than consumption public spending ($G_{c.i,t}$), and more efficient in increasing the productivity of private factors, which implies: $(0 < z_2 < z_1 < 1)$, whereas: $(G_{i,t} = G_{inv.i,t} + G_{c.i,t})$. Indeed, the consequences of an increase of the public indebtedness level would likely be different if this public debt is only intended to finance current consumption public expenditure or if it allows public investment in capital. Therefore, the production function for the representative firm in the country (i) is:

$$Y_{i,t} = A_{i,t}(K^d_{ii,t} + K^d_{ji,t})^\nu \left(L^d_{i,t}\right)^{1-\nu} G^{z_1}_{inv.i,t} G^{z_2}_{c.i,t} \quad (11)$$

and in terms of variations:

$$y_{i,t} = a_{i,t} + \nu k^d_{.i,t} + (1-\nu)l^d_{i,t} + z_1 g_{inv.i,t} + z_2 g_{c.i,t} \quad (11')$$

With, for the country (i) in period (t): ($A_{i,t}$): technology or productivity shock, common to all firms in the country, or Total Factor Productivity; ($Y_{i,t}$): real production level; ($K^d_{ji,t}$): capital demanded to households in the country (j) from firms in the country (i).

Non growth enhancing public expenditure ($G_{c.i,t}$) is, for example: justice, defense or social welfare, whereas growth enhancing public expenditure ($G_{inv.i,t}$) is: public infrastructure, education or health.

This firm maximizes its nominal profit: $\Pi_{i,t} = P_{i,t}Y_{i,t} - R_t K^d_{.i,t} - W_{i,t}L^d_{i,t}$.

Production factors are paid at their marginal products, and we suppose constant returns. So, the maximization of the profit and equation (11) imply:

$$P_{i,t}\frac{\partial Y_{i,t}}{\partial K^d_{.i,t}} = \nu A_{i,t}P_{i,t}(K^d_{.i,t})^{-(1-\nu)}\left(L^d_{i,t}\right)^{1-\nu} G^{z_1}_{inv.i,t} G^{z_2}_{c.i,t} = R_t > 0 \quad ; \quad \frac{R_t K^d_{.i,t}}{P_{i,t}Y_{i,t}} = \nu \quad (12)$$

$$P_{i,t}\frac{\partial Y_{i,t}}{\partial L^d_{i,t}} = (1-\nu)A_{i,t}P_{i,t}(K^d_{.i,t})^{\nu}\left(L^d_{i,t}\right)^{-\nu} G^{z_1}_{inv.i,t} G^{z_2}_{c.i,t} = W_{i,t} > 0 \quad ; \quad \frac{W_{i,t}L^d_{i,t}}{P_{i,t}Y_{i,t}} = (1-\nu) \quad (13)$$

Therefore, by combining equations (12) and (13), we obtain the following relation between the nominal wage in the country (i) and the common nominal interest rate:

$$W_{i,t} = \frac{(1-\nu)K^d_{.i,t}}{\nu L^d_{i,t}} R_t \quad (14)$$

Besides, households are free to invest their capital wherever they want. So, assuming rational behavior, capital moves across borders to seek the highest net-of-tax return. Allocation is then defined according to post-tax rates of return, which are equated across countries. When the representative consumer choses his investment, the profitability of private firms decreased by the taxation rate on this profitability must be the same across all firms in the member countries of the monetary union, and equal to the world net of tax real capital return (ρ) on the world financial market. So, the 'capital arbitrage condition' is:

$$\frac{R_t}{P_{i,t}}\left(1-t_{i,t}^k\right)=\frac{R_t}{P_{j,t}}\left(1-t_{j,t}^k\right)=\rho_t \tag{15}$$

which also implies: $$\left(r_t-\pi_{i,t}\right)=-\ln\left(\frac{1-t_{i,t}^k}{1-t_{i,t-1}^k}\right)+\ln\left(\frac{\rho_t}{\rho_{t-1}}\right) \tag{16}$$

Therefore, the required before-tax real interest rate must increase in proportion to the capital taxation rate, in order to preserve a same post-tax real return for a capital investment. It must also increase with the world net of tax real capital return.

Besides, we consider a Calvo-type framework of staggered prices, where a fraction ($0<\alpha<1$) of goods prices remain unchanged each period, whereas prices are adjusted for the remaining fraction ($1-\alpha$) of goods. The optimal strategy of the firm is to fix reset prices at marginal costs. In this case, we can show that the inflation rate in the country (i) in period (t) verifies the following equation:

$$\pi_{i,t}=\beta E_t\left(\pi_{i,t+1}\right)+\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[\ln\left(MC_{i,t}\right)-\ln\left(P_{i,t}\right)\right] \tag{17}$$

where ($MC_{i,t}$) is the nominal marginal production cost of the firm.

Equations (12) and (13) give the nominal production costs ($W_{i,t}L_{i,t}^d$) and ($R_tK_{.i,t}^d$) for the production of the quantity ($Y_{i,t}$) for a representative firm in the country (i). So, by differentiating these equations, the nominal marginal production cost of the quantity ($Y_{i,t}$) is:

$$MC_{i,t}=\frac{\partial\left(W_{i,t}L_{i,t}^d\right)}{\partial Y_{i,t}}+\frac{\partial\left(R_tK_{.i,t}^d\right)}{\partial Y_{i,t}}=(1-\nu)P_{i,t}+\nu P_{i,t}=P_{i,t} \tag{18}$$

Therefore, equations (17) and (18) imply that the optimal inflation rate is simply:

$$\pi_{i,t}=\beta E_t\left(\pi_{i,t+1}\right) \tag{19}$$

The current inflation rate then only depends on expected future inflation.

3.3 Budgetary policy and public debt level

The levels of public expenditure, taxation rates and public debt are fixed at the national level by the budgetary authorities. For simplicity, we suppose that all government debt is held domestically, and is risk free real debt, in conformity with most empirical observations. The government is supposed to be able to credibly commit to repay the public debt. For the government of the country (i), the budgetary constraint is then the following:

$$\begin{aligned}B_{i,t}=(1+R_{t-1})B_{i,t-1}+P_{i,t}\left(G_{inv.i,t}+G_{c.i,t}\right)-t_{i,t}^cP_{i,t}C_{i,t}-t_{i,t}^lW_{i,t}L_{i,t}\\ -t_{i,t}^k(R_t-\delta P_{i,t})(K_{ii,t}+K_{ui,t})\end{aligned} \tag{20}$$

The public debt of the country (i) in period (t) equals the public debt of the former period (t-1) increased by the interest rates on this previous public debt, plus the public consumption and investment expenditure to be financed, decreased by fiscal resources of the current period. In a source-based taxation system, the latter include consumption and labor taxation, and capital taxation on national and foreign capital invested in the national country, taking into account that physical capital depreciation is exempted from taxation.

Using equations (15) and (20) and the constant respective shares of the production factors in equations (12) and (13), the real public debt in proportion of real GDP ($P_{i,t}Y_{i,t}$), is:

$$\begin{aligned}\left(\frac{B_{i,t}}{P_{i,t}Y_{i,t}}\right)=\frac{(1+R_{t-1})}{\left(1+\pi_{i,t}\right)\left(1+y_{i,t}\right)}\left(\frac{B_{i,t-1}}{P_{i,t-1}Y_{i,t-1}}\right)+\frac{\left(G_{inv.i,t}+G_{c.i,t}\right)}{Y_{i,t}}-\frac{t_{i,t}^cC_{i,t}}{Y_{i,t}}\\ -(1-\nu)t_{i,t}^l-\frac{\nu\left(\rho_t-\delta+\delta t_{i,t}^k\right)t_{i,t}^k}{\rho_t}\end{aligned} \tag{21}$$

Solving equation (21) forwards, using equation (4), with $\lim_{t\to\infty}B_{i,t}=0$ (we suppose a no-Ponzi-game, and the satisfaction of the intertemporal budgetary constraint), we can obtain:

$$\left(\frac{B_{i,t}}{P_{i,t}Y_{i,t}}\right)=\beta E_t\{\sum_{n=0}^{\infty}\beta^n\frac{\left(1+y_{i,t+n+1}\right)\dots\left(1+y_{i,t+1}\right)\left(1+t_{i,t}^c\right)C_{i,t}}{\left(1+t_{i,t+n+1}^c\right)C_{i,t+n+1}}\ [(1-\nu)t_{i,t+n+1}^l$$

$$+\frac{\nu\left(\rho_{t+n+1}-\delta+\delta t^k_{i,t+n+1}\right)}{\rho_{t+n+1}}t^k_{i,t+n+1}-\frac{\left(G_{inv.i,t+n+1}+G_{c.i,t+n+1}\right)}{Y_{i,t+n+1}}+t^c_{i,t+n+1}\frac{C_{i,t+n+1}}{Y_{i,t+n+1}}]\} \quad (22)$$

Appendix 1 details the expressions of the long run equilibrium values of all components of global demand of our macro-economic model[5].

Besides, log-linearizing equation (21), as $(\pi_{i,t})$ $(y_{i,t})$ and (R_t) are small, and using equation (8), in variation from its long term equilibrium value, we obtain:

$$\widehat{b_{\iota,t}}=\frac{(1+R)}{(1+\pi_i)(1+y_i)}\left(\widehat{b_{\iota,t-1}}+R_{t-1}-R_{t-2}-\pi_{i,t}+\pi_{i,t-1}-y_{i,t}+y_{i,t-1}\right)$$
$$+\frac{P_iG_i}{B_i}(c_{i,t}-y_{i,t})-\frac{t^c_iP_iC_i}{B_i}\left[c_{i,t}-y_{i,t}+\ln\left(\frac{t^c_{i,t}}{t^c_{i,t-1}}\right)\right]$$
$$-\frac{(1-\nu)t^l_iP_iY_i}{B_i}\ln\left(\frac{t^l_{i,t}}{t^l_{i,t-1}}\right)-\frac{\nu\left(\rho-\delta+\delta t^k_i\right)t^k_iP_iY_i}{\rho B_i}\ln\left[\frac{\rho_{t-1}\left(\rho_t-\delta+\delta t^k_{i,t}\right)t^k_{i,t}}{\rho_t\left(\rho_{t-1}-\delta+\delta t^k_{i,t-1}\right)t^k_{i,t-1}}\right] \quad (23)$$

Where $(\widehat{b_{\iota,t}})$ is the variation of the real debt to GDP ratio.

Therefore, using equations (23), (A1.6) and (A1.7) in Appendix 1, in order to avoid an outbidding of the public debt (B_i=0), economic variables must verify the following system:

$$\begin{cases} \widehat{b_{\iota,t}}=\frac{(1+R)}{(1+\pi_i)(1+y_i)}\left(\widehat{b_{\iota,t-1}}+R_{t-1}-R_{t-2}-\pi_{i,t}+\pi_{i,t-1}-y_{i,t}+y_{i,t-1}\right) \\ \rho\frac{(G_i-t^c_iC_i)}{Y_i}(c_{i,t}-y_{i,t})-\rho\frac{t^c_iC_i}{Y_i}\ln\left(\frac{t^c_{i,t}}{t^c_{i,t-1}}\right) \\ =\rho(1-\nu)t^l_i\ln\left(\frac{t^l_{i,t}}{t^l_{i,t-1}}\right)+\nu\left(\rho-\delta+\delta t^k_i\right)t^k_i\ln\left[\frac{\rho_{t-1}\left(\rho_t-\delta+\delta t^k_{i,t}\right)t^k_{i,t}}{\rho_t\left(\rho_{t-1}-\delta+\delta t^k_{i,t-1}\right)t^k_{i,t-1}}\right] \end{cases} \quad (24)$$

3.4 The global equilibrium

We are now going to derive the equilibrium on the goods market regarding global demand. Clearing on the goods market, equality between supply and demand of goods and services, in the country (i) in period (T) requires:

$$Y_{i,T}=C_{i,T}+G_{inv.i,T}+G_{c.i,T}+(INV_{ii,T}+INV_{ji,T})+EX_{i,T} \quad (25)$$

With ($EX_{i,T}$): net exports of the country (i).

Furthermore, in variation from the last period's value, equation (25) also implies:

$$y_{i,T}=\frac{C_i}{Y_i}c_{i,T}+\frac{G_i}{Y_i}g_{i,T}+\frac{INV_{.i}}{Y_i}inv_{.i,T}+\frac{EX_i}{Y_i}ex_{i,T} \quad (26)$$

Where the relative shares of the components of global demand are mentioned in Appendix 1.

By combining equations (5) and (24), and using footnote 5, we obtain:

$$y_{i,t}=E_t\left(y_{i,t+1}\right)-\left[R_t-R_{t-1}-E_T\left(\pi_{i,t+1}\right)+\pi_{i,t}-\overline{R_{\iota,t}}+\overline{R_{\iota,t-1}}\right] \quad (27)$$

$$with:\ \overline{R_{\iota,t}}=\frac{\rho(1-\nu)t^l_i}{\left[\rho\left(t^l_i+\nu t^k_i-\nu t^l_i\right)-\nu\delta t^k_i\left(1-t^k_i\right)\right]}E_t\left[\ln\left(\frac{t^l_{i,t+1}}{t^l_{i,t}}\right)\right]$$
$$+\frac{\rho t^c_iC_i}{\left[\rho\left(t^l_i+\nu t^k_i-\nu t^l_i\right)-\nu\delta t^k_i\left(1-t^k_i\right)\right]Y_i}E_t\left[\ln\left(\frac{t^c_{i,t+1}}{t^c_{i,t}}\right)\right]+E_t\left[\ln\left(\frac{1+t^c_{i,t+1}}{1+t^c_{i,t}}\right)\right]$$
$$+\frac{\nu\left(\rho-\delta+\delta t^k_i\right)t^k_i}{\left[\rho\left(t^l_i+\nu t^k_i-\nu t^l_i\right)-\nu\delta t^k_i\left(1-t^k_i\right)\right]}E_t\{\ln\left[\frac{\rho_t\left(\rho_{t+1}-\delta+\delta t^k_{i,t+1}\right)t^k_{i,t+1}}{\rho_{t+1}\left(\rho_t-\delta+\delta t^k_{i,t}\right)t^k_{i,t}}\right]\}$$

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[5] Equations (A1.2) and (A1.3) imply: $\left(\frac{G_i}{Y_i}-t^c_i\frac{C_i}{Y_i}\right)=\frac{\left[\rho\left(t^l_i+\nu t^k_i-\nu t^l_i\right)-\nu\delta t^k_i\left(1-t^k_i\right)\right]}{\rho}$.

With equation (22), we can demonstrate that the long run value for the real debt to GDP ratio is: $\left(\frac{B_i}{P_iY_i}=0\right)$.

$(\overline{R_{\iota,T}})$ is the equilibrium or natural real interest rate in the country (i), which corresponds to the steady-state real rate of return if prices and wages were fully flexible. It is the real interest rate required to keep aggregate demand equal at all times to the natural rate of output. It includes all non-monetary disturbances. It is an increasing function of the anticipated increase of the labor, capital or consumption taxation rates.

So, according to equation (27), a higher future expected output increases current output and consumption, because households prefer to smooth consumption, and then higher future revenues raise their current consumption and current output. Current output is also a decreasing function of the excess of the real interest rate above its natural level, because of the inter-temporal substitution of consumption.

Besides, equations (12) and (15) imply:

$$\frac{INV_{.i,T}}{Y_{i,T}} = \left(\frac{INV_{.i,T}}{K_{.i,T}}\right)\left(\frac{K_{.i,T}}{Y_{i,T}}\right) = \delta\frac{\nu P_{i,T}}{R_T} = \frac{\nu\delta\left(1 - t_{i,t}^k\right)}{\rho_t} \tag{28}$$

So, we have the following components of global demand. First, equation (28) implies:

$$inv_{i,t} = y_{i,t} + ln\left(\frac{1 - t_{i,t}^k}{1 - t_{i,t-1}^k}\right) - \ln\left(\frac{\rho_t}{\rho_{t-1}}\right) \tag{29}$$

Equations (24) and (8) imply:

$$c_{i,t} = g_{i,t} = y_{i,t} + \overline{R_{\iota,t-1}} - \ln\left(\frac{1 + t_{i,t}^c}{1 + t_{i,t-1}^c}\right) \tag{30}$$

If we note $(\varepsilon_{i,t}^{g,inv})$ the shock on public investment, equation (30) then implies:

$$g_{inv.i,t} = y_{i,t} + \varepsilon_{i,t}^{g,inv} \tag{31}$$

$$g_{c.i,t} = \frac{G_i}{G_{c.i}} g_{i,t} - \frac{G_{inv.i}}{G_{c.i}} g_{inv.i,t} = y_{i,t} + \frac{G_i}{G_{c.i}}\overline{R_{\iota,t-1}} - \frac{G_{inv.i}}{G_{c.i}}\varepsilon_{i,t}^{g,inv} - \frac{G_i}{G_{c.i}}\ln\left(\frac{1 + t_{i,t}^c}{1 + t_{i,t-1}^c}\right) \tag{32}$$

Finally, equations (26), (29), (30), (31) and (32) imply:

$$ex_{i,t} = y_{i,t} + \frac{(C_i + G_i)}{EX_i}\ln\left(\frac{1 + t_{i,t}^c}{1 + t_{i,t-1}^c}\right) - \frac{(C_i + G_i)}{EX_i}\overline{R_{\iota,t-1}} - \frac{INV_{.i}}{EX_i}\left[ln\left(\frac{1 - t_{i,t}^k}{1 - t_{i,t-1}^k}\right) - \ln\left(\frac{\rho_t}{\rho_{t-1}}\right)\right] \tag{33}$$

3.5 Calibration of the parameters

We consider a standard calibration of the parameters of our model, in conformity with the economic literature. However, in the following section 4 of the paper, we will mention and analyze the sensibility of our results to variations in these parameters.

The EUTAX model of Sorensen (2001) calibrates the share of capital in the production function at $(\nu = 0.33)$, whereas Mendoza (2001) or Mendoza and Tesar (2005) calibrate this share at (ν=0.36). In conformity with empirical data and with economic studies, we can then calibrate the share of capital in the production function at (ν=0.33). Besides, according to empirical data, the world after tax real capital return can be calibrated around (ρ=0.08). Mendoza and Tesar (2005) calibrate the depreciation rate of capital at (δ=0.02); we will retain a value close to the one mentioned in most economic studies: (δ=0.025).

In conformity with implicit tax rates mentioned by the European Commission (2018), we will consider the following average long run taxation rates regarding: capital $(t_i^k = 0.3)$, labor $(t_i^l = 0.2)$ and consumption $(t_i^c = 0.15)$. According to the 'capital arbitrage condition' [see equation (15)], real capital returns are equalized across countries; therefore, these are only differences in size, or location rents (in terms of high skill workers, developed infrastructure), which can explain cross countries investments in capital. Then, we make the hypothesis that foreign source capital is quite negligible in comparison with national capital: $(\frac{K_{ij}}{K_{ii}} = \frac{K_{ji}}{K_{ii}} = 0.05)$. Therefore, according to equations in Appendix 1, taken together, these calibrated parameters correspond to the following levels of long term components of global demand: $(\frac{C_i}{Y_i} = 0.56)$, $(\frac{G_i}{Y_i} = 0.30)$, $(\frac{INV_{.i}}{Y_i} = \frac{EX_i}{Y_i} = 0.07)$. These values are not too far from empirical values mentioned in the economic literature.

The preference for the present (β) is usually calibrated at (β=0.99) in the economic literature. The calibration of the inverse of the Frisch labor supply elasticity (φ) is very heterogeneous in the economic literature, going from (0.2) in Gali *et al.* (2007), until (2) in Coenen and Straub (2005) or in Leeper *et al.* (2011). In this paper, we will consider (φ=1). The productivity of public consumption expenditure is estimated around (z_2=0.05) in Sims and Wolff (2013) or in Carvalho and Martins (2011), whereas the productivity of public capital investment (highly productive) is (z_1=0.16) in Carvalho and Martins (2011).

We will consider the following value of the nominal capital return in the European Economic and Monetary Union: (R=0.08). We will consider a nominal equilibrium growth rate around (y_i=0.03), and a long term equilibrium inflation rate around the European Central Bank target: (π_i=0.02). Finally, we will consider that among public expenditure, public investment is much below public consumption expenditure, and therefore, that ($\frac{G_{inv.i}}{G_{c.i}} = 0.2$).

4 Consequences of the public debt level for economic growth

In the framework of the previous theoretical model, we can now detail the dimensions by which the growth of current and previous public debt levels are liable to influence global economic activity, the various components of global demand, as well as the equilibrium on the labor market (see Appendices 2 and 3), for a member country of a monetary union. Equation (A2.8) shows that the inflation rate should be constant in the monetary union. But what could be the consequences of the increase of the current public debt level on global economic activity and on the various factors of global demand?

4.1 Increase of current public debt and various factors of global demand

According to equation (A3.2) in Appendix A3, without any variation of taxation rates in order to finance the public debt, if [$\frac{\partial \overline{R_{\iota,t-1}}}{\partial(b_{i,t}-\pi_{i,t})}$=0], the public indebtedness strongly and proportionately increases economic activity [$\frac{\partial y_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = (2-\beta)$]. But the mechanism in our model implies that this debt must be financed with an increase of future taxation rates, in order to avoid an outbidding of the public indebtedness level. So, according to equation (A3.4), the public debt has an ambiguous effect. The increase of the current public debt provides additional fiscal resources; it can reduce the equilibrium interest rate and current capital, labor or consumption taxation rates [see $\overline{R_{\iota,t-1}}$ in equation (27)]. However, this public debt will have to be repaid, and so, it increases future expected taxation rates. Let's mention that the previous reasoning supposes that: $(R - y_i - \pi_i - \pi_i y_i) > 0$, and therefore that in the long term, the nominal capital return (or nominal interest rate) is above the nominal growth rate; capital asset owners enjoy higher rates of wealth growth than do labor asset owners.

Besides, equations (29), (30), (33), (A3.4) and (A3.5) imply that an increase of the consumption taxation rate ($t_{i,t}^c$) in comparison with the previous period obviously decreases private consumption, whereas it increases net exports. In the same way, an increase of the capital taxation rate ($t_{i,t}^k$) or of the world capital return (ρ_t) in comparison with the previous period reduces national private investment, whereas it increases net exports.

However, we are mainly interested in this paper with the effect of a variation of the public debt level. Therefore, regarding the latter, the previously mentioned equations as well as equations (8) and (A3.7) imply:

$$\frac{\partial y_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = \frac{\partial inv_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = \frac{\partial k_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 1 + \frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}\left[1 - \frac{\partial R_{t-1}}{\partial\left(b_{i,t}-\pi_{i,t}\right)}\right] \quad (34)$$

$$\frac{\partial c_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = \frac{\partial g_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 1 + (1-\beta)\left[1 - \frac{\partial R_{t-1}}{\partial\left(b_{i,t}-\pi_{i,t}\right)}\right] \quad (35)$$

$$\frac{\partial ex_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 1 + \frac{(1-\beta)[(1+R)EX_i + (1+\pi_i)(1+y_i)(C_i+G_i)]}{(R-y_i-\pi_i-\pi_i y_i)EX_i}\left[1-\frac{\partial R_{t-1}}{\partial(b_{i,t}-\pi_{i,t})}\right] \quad (36)$$

So, an increase of the current public debt can mostly sustain net exports [$\frac{\partial ex_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 5.60$ with our basic calibration in section 3.5]. It also increases private investment [$\frac{\partial inv_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 1.37$], as well as public and private consumption [$\frac{\partial c_{i,t}}{\partial(b_{i,t}-\pi_{i,t})} = \frac{\partial g_{i,t}}{\partial(b_{i,t}-\pi_{i,t})}$=1.01]. More precisely, according to equations (37) and (38) below, public investment expenditure is favored [$\frac{\partial g_{inv.i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 4.91$] and increases more than public consumption expenditure [$\frac{\partial g_{c.i,t}}{\partial(b_{i,t}-\pi_{i,t})} = 0.23$]. Therefore, our theoretical model shows the potential beneficial short term effects of a higher public indebtedness level, mainly regarding the increase of public investment expenditure and of net exports.

More precisely, an increase of the current public debt implies an immediate increase of all components of economic activity, and all the more as the preference for the present is high (except for the consumption public expenditure). If economic agents are more impatient and favor the present, the current public debt strongly increases net exports, and then public investment expenditure. For example, according to our basic calibration, an increase of 1% of the current public debt would increase the economic activity level by 19.4% if (β=0.5), but only by 1.37% if (β=0.99) and if economic agents highly value the future.

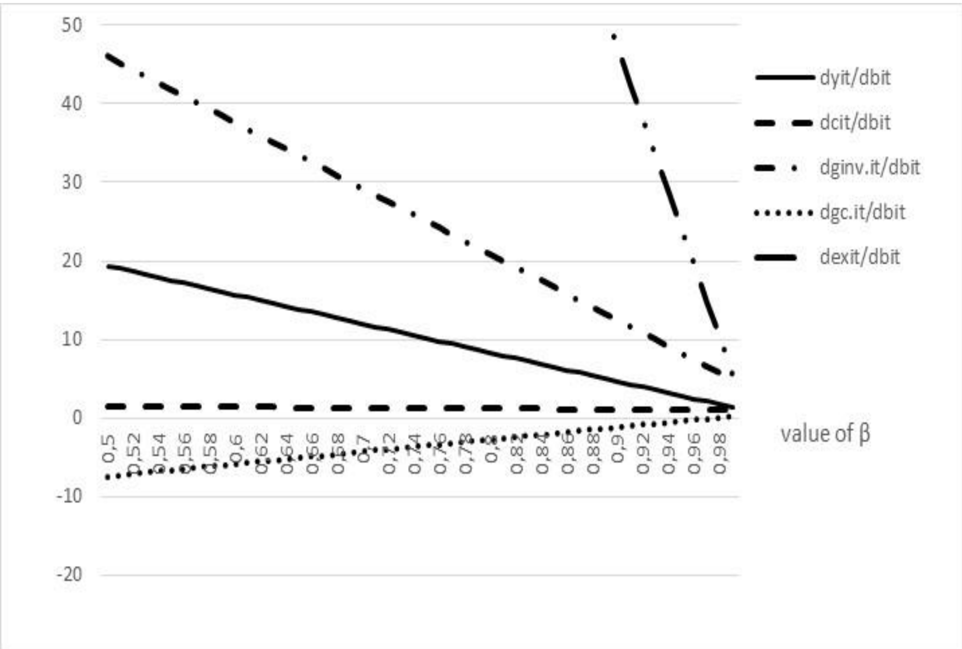


Figure 1: Effect of an increase of the current public debt on factors of global demand, according to the time discount factor (β)

Besides, an increase of the public debt is also more useful to sustain current economic activity, public and private investment and net exports if the long term nominal interest rate (R) is weak, and if the long term nominal economic growth rate ($y_i + \pi_i$) is high (see Figure 2). Indeed, the cost of the reimbursement of interests on the past public debt is then weaker.

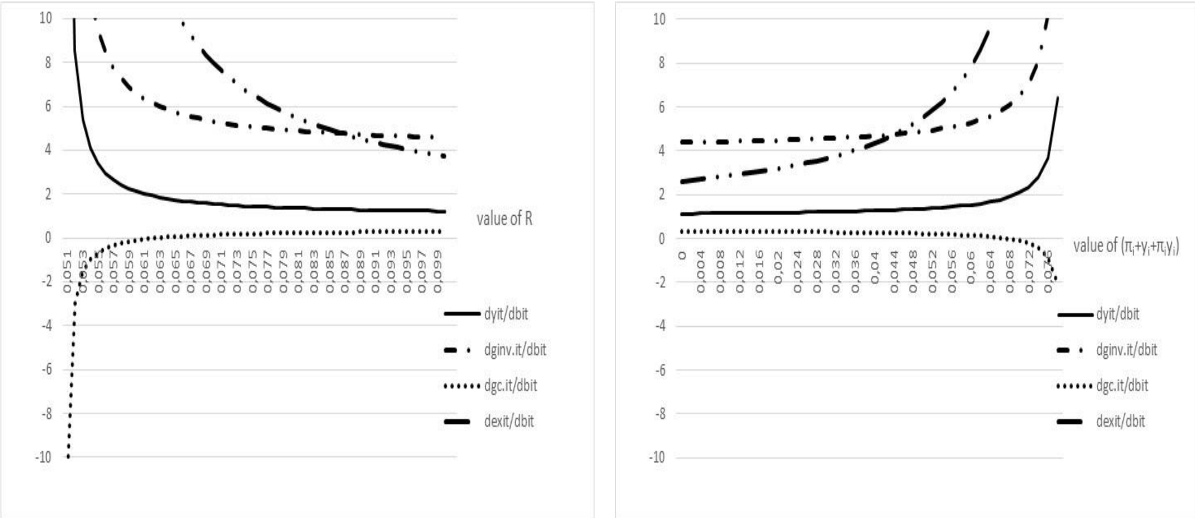


Figure 2: Effect of an increase of the current public debt on factors of global demand, according to the long term nominal interest rate (R) and economic growth rate (y_i+π_i)

Calibration: $(y_i + \pi_i + \pi_i y_i = 0.0506)$ (β=0.99) (R=0.08) (β=0.99)

Furthermore, we can mention that equilibrium and long term ratios of the components of global demand impact net exports, according to equation (33). So, if the capital share in the production function (ν) or if the capital depreciation rate (δ) are weaker, if the world capital return (ρ) is higher, if net capital exports $(K_{ij} - K_{ji})$ or if the equilibrium capital taxation rate (t_i^k) are higher in a given country (i), the share of nets exports in its global demand is smaller. So, after an increase of the current public debt level in this country (i), the increase of its net exports in comparison with the previous period would be accentuated, in order to maintain a same contribution of exports to the increase of global demand in this country.

4.2 Increase of current public debt and components of public expenditure

Regarding the components of public expenditure, an increase of the labor $(t_{i,t}^l)$, capital $(t_{i,t}^k)$ or consumption $(t_{i,t}^c)$ taxation rates, or an increase of the world real capital return (ρ_t), increases public investment and decreases public consumption expenditure, even if global public expenditure remains unchanged. On the contrary, an exogenous productivity ($a_{i,t}$) shock [see $(\varepsilon_{i,t}^{g,inv})$ in equations (31), (32) and (A3.12)] increases public consumption expenditure, and decreases public investment expenditure.

Regarding the consequences of an increase of the current public debt, with an exogenously defined long term ratio of public investment in comparison with consumption expenditure $(\frac{G_{inv.i}}{Gc,i})$, equations (31), (32), (A3.4), (A3.5) and (A3.12) imply:

$$\frac{\partial g_{inv.i,t}}{\partial(b_{i,t} - \pi_{i,t})} = \frac{\varphi(1-\nu)G_{c.i}(1-\beta)(1+\pi_i)(1+y_i)}{(z_1 G_{c.i} - z_2 G_{inv.i})(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left[1 - \frac{\partial R_{t-1}}{\partial(b_{i,t} - \pi_{i,t})}\right] + \frac{[(1-\nu)G_{c.i} - z_2(G_{c.i} + G_{inv.i})]}{(z_1 G_{c.i} - z_2 G_{inv.i})}\left[(2-\beta) - (1-\beta)\frac{\partial R_{t-1}}{\partial(b_{i,t} - \pi_{i,t})}\right] \quad (37)$$

$$\frac{\partial g_{c.i,t}}{\partial(b_{i,t} - \pi_{i,t})} = -\frac{\varphi(1-\nu)G_{inv.i}(1-\beta)(1+\pi_i)(1+y_i)}{(z_1 G_{c.i} - z_2 G_{inv.i})(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left[1 - \frac{\partial R_{t-1}}{\partial(b_{i,t} - \pi_{i,t})}\right] - \frac{[(1-\nu)G_{inv.i} - z_1(G_{c.i} + G_{inv.i})]}{(z_1 G_{c.i} - z_2 G_{inv.i})}\left[(2-\beta) - (1-\beta)\frac{\partial R_{t-1}}{\partial(b_{i,t} - \pi_{i,t})}\right] \quad (38)$$

Therefore, regarding the destination of the fiscal resources provided by a higher public indebtedness level, our model shows that they are mainly devoted to increase the share of public investment expenditure. Furthermore, the higher current public debt would be more useful to sustain public investment expenditure to the detriment of public consumption expenditure if the Frisch labor supply elasticity ($1/\varphi$) is weak, or if (z_1/z_2) or if (z_2) are small. Indeed, public investment expenditure is then weakly productive and relatively less advantageous than public consumption expenditure, and it must thus be accentuated for a same variation of global public expenditure and for a same long term ratio ($\frac{G_{inv.i}}{Gc,i}$). If public expenditure is weakly productive, or if public investment expenditure is less substantially productive than public consumption expenditure, productive public investment must increase more in comparison with public consumption expenditure, in order to allow the same increase of global economic activity. Public consumption expenditure can then even decrease, despite the higher current public debt level.

A higher current public debt is also more useful to increase the relative share of public investment expenditure if the desired relative long term share of this public investment ($\frac{G_{inv.i}}{Gc,i}$) is higher. Indeed, in order to reach a higher long term share of public investment expenditure, its growth should be accentuated. On the contrary, the long term share of public consumption expenditure is weaker; its growth should then be reduced, and it could even decrease in order to provide the same growth of global public expenditure.

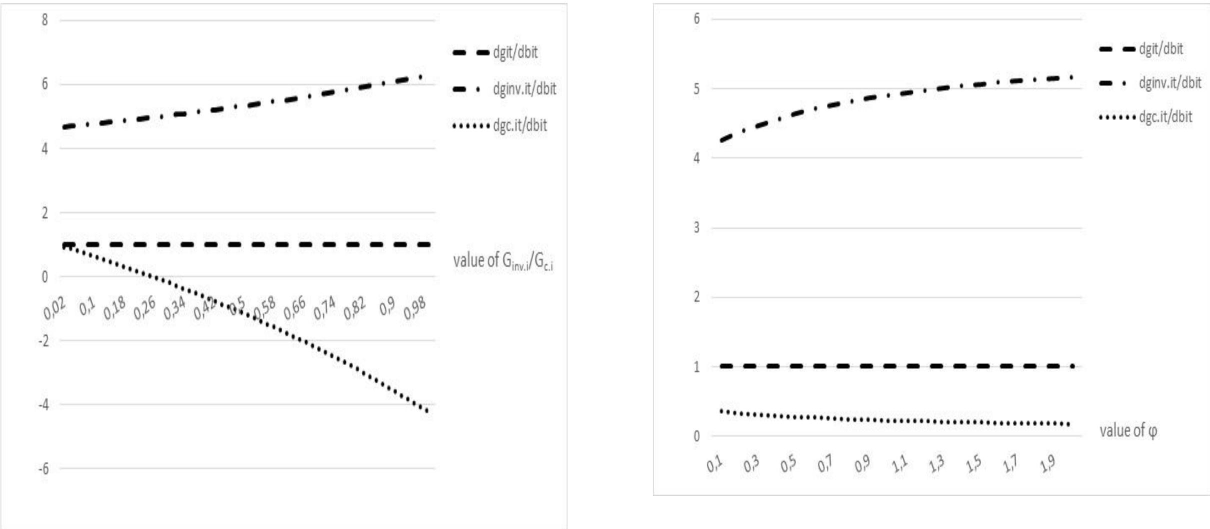
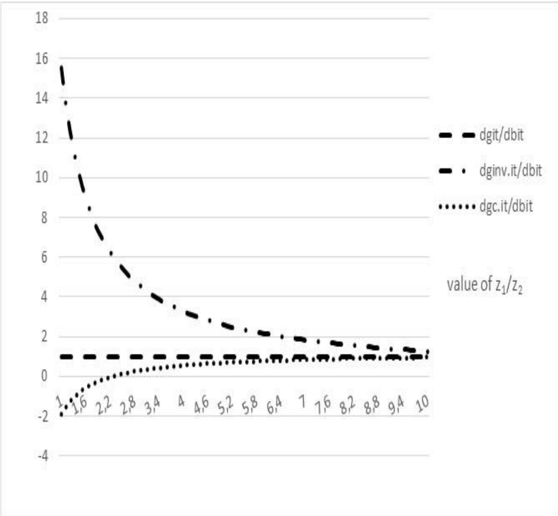
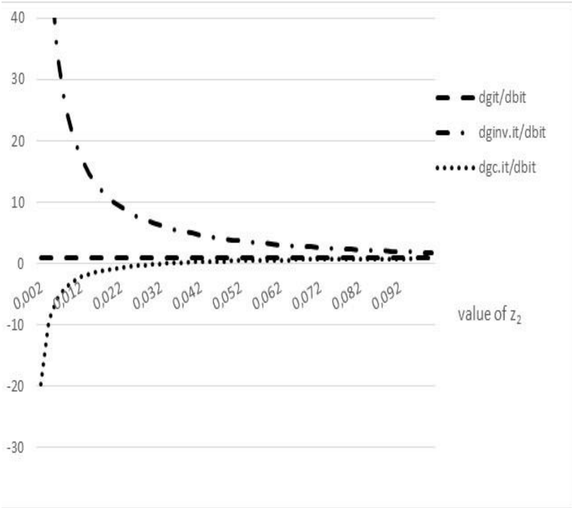


Figure 3: Effect of an increase of the current public debt on the components of public expenditure, according to their productivity (z), to the long term ratio of public investment and consumption expenditure ($\frac{G_{inv.i}}{Gc,i}$), to the Frisch labor supply elasticity ($1/\varphi$)

Calibration : $(z_1 = 0.16)$ $(z_2 = 0.05)$ (φ=1) $(z_1 = 0.16)$ $(z_2 = 0.05)$ ($\frac{G_{inv.i}}{G_{c.i}} = 0.2$)





Calibration : ($z_2 = 0.05$) ($\frac{G_{inv.i}}{G_{c.i}} = 0.2$) (φ=1) ($\frac{z_1}{z_2} = 4$) ($\frac{G_{inv.i}}{G_{c.i}} = 0.2$) (φ=1)

Finally, after an increase of the current public debt, if the capital share in the production function (ν) is weak, the share of public investment expenditure increases, while the share of public consumption expenditure decreases, for a same increase of global public expenditure.

4.3 Increase of the current public debt and the labor market

After studying the market for goods and services, what could be the consequences of the growth of the current public indebtedness level on the labor market?

According to equations (A3.7) and (16), the real capital return in the country (i) ($r_t - \pi_{i,t}$) increases while the capital stock ($k_{i,t}$) decreases with the increase of the real world capital return (ρ_t) or of the national capital taxation rate ($t_{i,t}^k$) in comparison with the previous period. Indeed, these equations imply:

$$\frac{\partial k_{i,t}}{\partial t_{i,t}^k} = -\frac{\partial(r_t - \pi_{i,t})}{\partial t_{i,t}^k} = -\frac{1}{(1 - t_{i,t}^k)} < 0 \tag{39}$$

$$\frac{\partial k_{i,t}}{\partial \rho_t} = -\frac{\partial(r_t - \pi_{i,t})}{\partial \rho_t} = -\frac{1}{\rho_t} < 0 \tag{40}$$

Besides, labor supply and demand decrease while the real wage increases with the increase of the labor taxation rate ($t_{i,t}^l$) in comparison with the previous period. Indeed, a higher labor taxation rate is disincentive for the work effort, while on the contrary, it necessitates a higher real wage in order to preserve the same purchasing power for households. So, equations (A3.10) and (A3.11) imply:

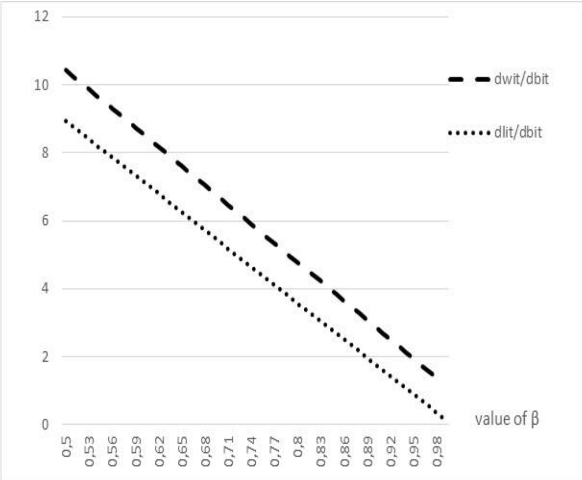
$$\frac{\partial l_{i,t}}{\partial t_{i,t}^l} = -\frac{\partial(w_{i,t} - \pi_{i,t})}{\partial t_{i,t}^l} = -\frac{1}{(1+\varphi)(1 - t_{i,t}^l)} < 0 \tag{41}$$

Regarding the consequences of the public indebtedness level on the labor market, we have shown in the previous section 4.1 that increasing the public debt is beneficial to current economic activity. According to equation (34), the capital stock then increases at the same pace as global economic activity. So, equations (A3.10) and (A3.11) imply:

$$\frac{\partial(w_{i,t} - \pi_{i,t})}{\partial(b_{i,t} - \pi_{i,t})} = 1 + \left[1 - \beta + \frac{\varphi(1-\beta)(1+\pi_i)(1+y_i)}{(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\right]\left[1 - \frac{\partial R_{t-1}}{\partial(b_{i,t} - \pi_{i,t})}\right] \tag{42}$$

$$\frac{\partial l_{i,t}}{\partial(b_{i,t} - \pi_{i,t})} = \frac{(1-\beta)(1+y_i)(1+\pi_i)}{(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left[1 - \frac{\partial R_{t-1}}{\partial(b_{i,t} - \pi_{i,t})}\right] \tag{43}$$

Therefore, the higher current public debt allows to increase mainly the real wage [$\frac{\partial(w_{i,t}-\pi_{i,t})}{\partial(b_{i,t}-\pi_{i,t})}=1.19$ with our basic calibration], whereas labor supply and demand increase more moderately [$\frac{\partial l_{i,t}}{\partial(b_{i,t}-\pi_{i,t})}=0.18$]. Furthermore, after an increase of the current public debt level, as for global economic activity, the real wage and labor supply and demand increase all the more in the short run as there is a high preference for the present (β is small) or as the long term nominal interest rate (R) is weak, whereas the long term nominal growth rate ($y_i+\pi_i$) is high. A higher Frisch labor supply elasticity (1/φ) also increases labor demand and supply, while it decreases the real wage.



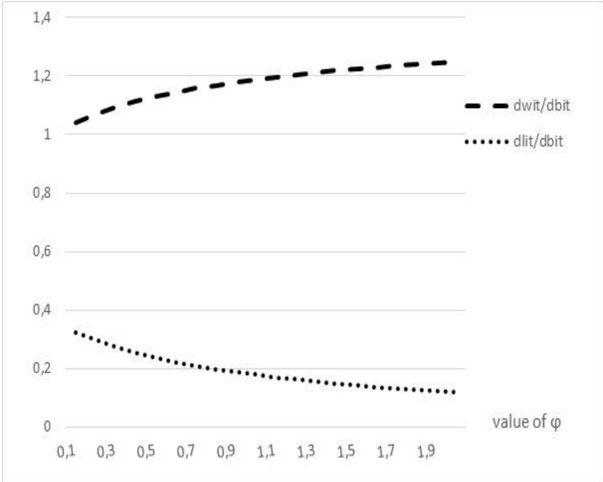
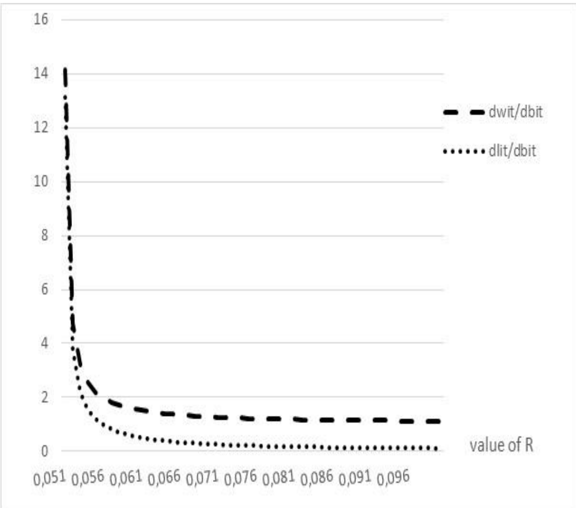
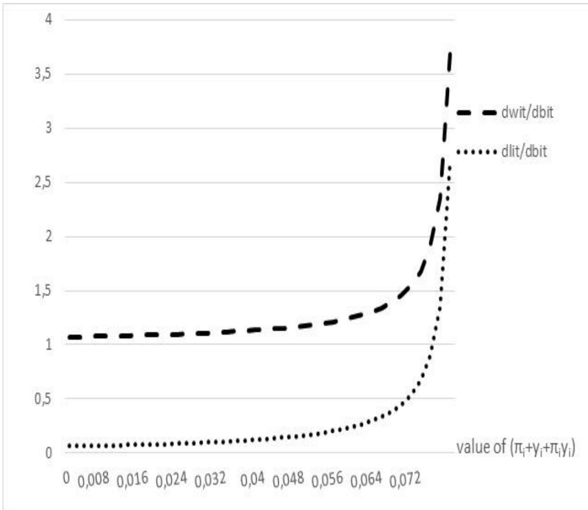


Figure 4: Effect of an increase of the current public debt on the real wage and on the labor factor, according to the time discount factor (β), to the Frisch labor supply elasticity (1/φ), to the long term nominal interest rate (R) and economic growth rate ($y_i+\pi_i$)

(φ=1) (R=0.08) ($y_i+\pi_i+\pi_i y_i=0.0506$) (β=0.99) (R=0.08) ($y_i+\pi_i+\pi_i y_i=0.0506$)





(β=0.99) (φ=1) ($y_i+\pi_i+\pi_i y_i=0.0506$) (β=0.99) (φ=1) (R=0.08)

5 Limits of the growth-enhancing effect of the public debt

The previous section 4 has shown that a higher public indebtedness level can contribute to increase the current economic activity level. However, our modelling can also underline some factors limiting or even preventing this growth-enhancing effect.

5.1 The former interest rate

In the framework of our model, an increase of the previous nominal interest rate (R_{t-1}), a more restrictive monetary policy, slightly decreases global economic activity. It is mostly detrimental to net exports [$\frac{\partial ex_{i,t}}{\partial R_{t-1}} = -4.60$ with our basic calibration]. It also weakly decreases private investment [$\frac{\partial inv_{i,t}}{\partial R_{t-1}} = -0.37$], public and private consumption [$\frac{\partial c_{i,t}}{\partial R_{t-1}} = \frac{\partial g_{i,t}}{\partial R_{t-1}} = -0.01$]. In the same way, the real wage and labor supply and demand also weakly decrease [$\frac{\partial (w_{i,t}-\pi_{i,t})}{\partial R_{t-1}} = -0.19$ and $\frac{\partial l_{i,t}}{\partial R_{t-1}} = -0.18$].

Therefore, we can see, in the previous equations (34) to (38), (42) and (43) that an increase of the nominal interest rate is detrimental to economic growth (except for public consumption expenditure). It reduces the short term growth-enhancing effect of a higher public debt level in all these equations. So, there is a first danger of a higher public indebtedness in the long run. If the public debt always increases from period and period, and if the absolute value of the global public debt level then becomes excessively high and unsustainable, financial markets can become reluctant to lend to a given country. So, there is a harmful effect of a higher public debt if the solvability of the latter is put into question, if the debt to GDP ratio is too high, because then, the nominal interest rate increases. Indeed, the nominal interest rate can be endogenously defined as an increasing function of the public indebtedness level; the cost of the reimbursement of the public debt is then higher, which is detrimental to economic growth.

For example, in 2007 and 2008, the nominal interest rate fixed by the European Central Bank was quite high, respectively 4.3% and 4.6%, in a framework of 'public debt crisis' in the member countries of the Euro Area. Therefore, in this context, the high public debt level in Italy (resp: 99.8% in 2007 and 102.4% of GDP in 2008) was a non-negligible barrier to economic growth in this country. Indeed, real GDP decreased by -1.05% in 2008, and by -5.48% in 2009. In the same way, in Greece, the public debt level was also particularly high: 103.1% in 2007, and 109.4% of GDP in 2008. Real GDP then strongly decreased in this country, by -4.30% in 2009, by -5.48% in 2010, and until -9.13% in 2011.

According to equation (34), the increase of the current public debt level is beneficial and has a positive effect on the current economic growth [$\frac{\partial y_{i,t}}{\partial (b_{i,t}-\pi_{i,t})} > 0$] as long as:

$$\frac{\partial R_{t-1}}{\partial \left(b_{i,t} - \pi_{i,t}\right)} < 1 + \frac{(R - y_i - \pi_i - \pi_i y_i)}{(1-\beta)(1+R)} \tag{44}$$

Therefore, we can demonstrate the existence of non-linearities regarding the consequences of the public debt on economic growth, as mentioned in the economic literature and in section 2. If the elasticity of the nominal interest rate to the increase of the public debt is weak, the public debt has a positive effect on current economic activity. However, if this elasticity is high, if the high public indebtedness level can put into question the sustainability of this public debt, and if it is considered as dangerous and excessive by the financial markets, the accentuated increase of the nominal interest rate implies a negative effect of a higher public debt level on current economic activity.

5.2 The previous increase of the public debt

In addition to the increase of the nominal interest rate, another detrimental effect relies on the negative consequence of the previous increase of the public debt. So, if the current public debt could be beneficial in the short term to economic activity (see section 4), it could also have a detrimental effect on long term economic growth.

Indeed, equations (8), (29), (30), (33), (A3.4), (A3.5) and (A3.7) imply:

$$\frac{\partial y_{i,t}}{\partial\left(b_{i,t-1}-\pi_{i,t-1}\right)}=\frac{\partial inv_{i,t}}{\partial\left(b_{i,t-1}-\pi_{i,t-1}\right)}=\frac{\partial k_{i,t}}{\partial\left(b_{i,t-1}-\pi_{i,t-1}\right)}=-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)} \quad (45)$$

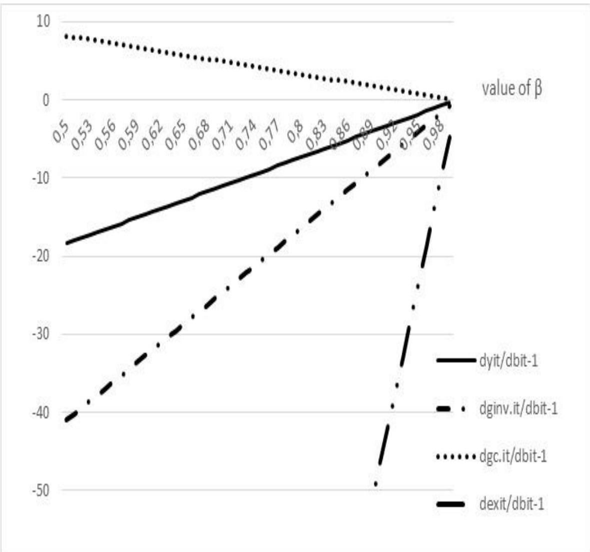
$$\frac{\partial ex_{i,t}}{\partial\left(b_{i,t-1}-\pi_{i,t-1}\right)}=-\frac{(C_i+G_i+EX_i)}{EX_i}\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)} \quad (46)$$

Therefore, the previous increase of the public debt is mainly detrimental to net exports [$\frac{\partial ex_{i,t}}{\partial(b_{i,t-1}-\pi_{i,t-1})}=-4.72$ with our basic calibration], if the long term nominal interest rate is above the long term nominal growth rate ($R>y_i+\pi_i+\pi_i y_i$). It also slightly decreases private investment [$\frac{\partial inv_{i,t}}{\partial(b_{i,t-1}-\pi_{i,t-1})}=-0.37$], global economic activity and the capital stock, without influencing private consumption or global public expenditure.

Furthermore, the previous increase of the public debt also reduces the real wage as well as labor demand and supply. Indeed, equations (A3.10) and (A3.11) imply:

$$\frac{\partial(w_{i,t}-\pi_{i,t})}{\partial\left(b_{i,t-1}-\pi_{i,t-1}\right)}=-\frac{\varphi(1-\beta)(1+R)}{(1+\varphi)(R-y_i-\pi_i-\pi_i y_i)}<0 \quad (47)$$

$$\frac{\partial l_{i,t}}{\partial\left(b_{i,t-1}-\pi_{i,t-1}\right)}=-\frac{(1-\beta)(1+R)}{(1+\varphi)(R-y_i-\pi_i-\pi_i y_i)}<0 \quad (48)$$



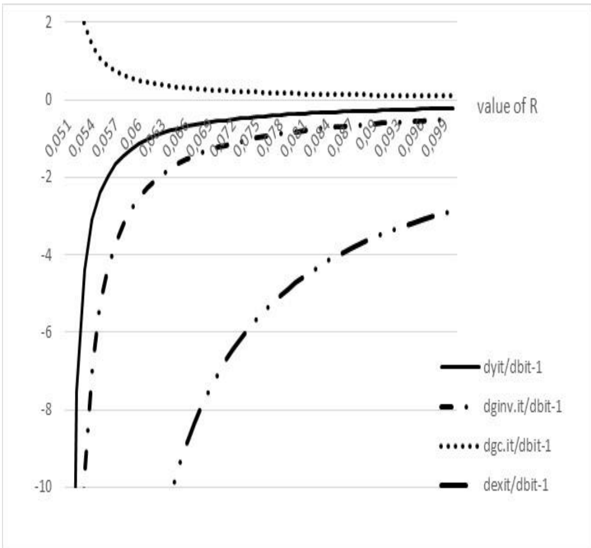
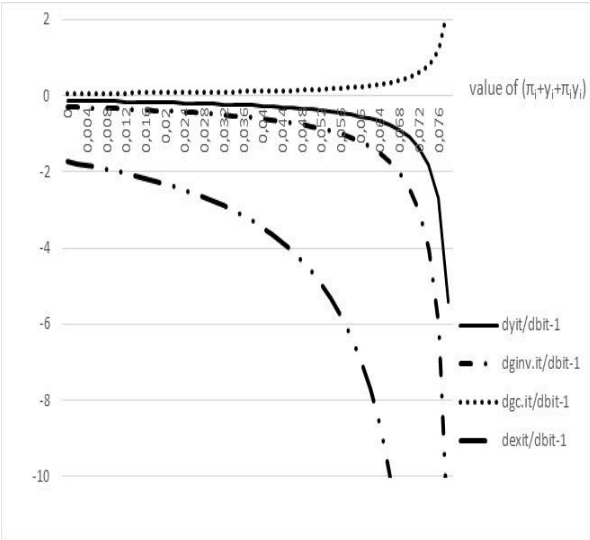
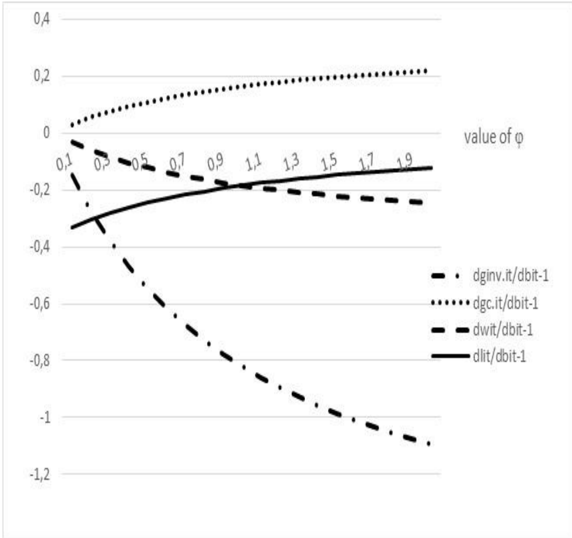


Figure 5: Effect of the previous increase of the public debt on the components of global demand, according to the time discount factor (β), to the Frisch labor supply elasticity (1/ φ), and to the long term nominal interest rate (R) and economic growth rate ($y_i+\pi_i$)

(φ=1) (R=0.08) ($y_i+\pi_i+\pi_i y_i=0.0506$) (β=0.99) (φ=1) ($y_i+\pi_i+\pi_i y_i=0.0506$)



(β=0.99) (φ=1) (R=0.08)



(β=0.99) (R=0.08) ($y_i + \pi_i + \pi_i y_i = 0.0506$)

Therefore, the previous increase of the public debt reduces economic activity (private and public investment and net exports), the real wage, labor supply and demand, even if it increases public consumption. Besides, this negative effect would be accentuated if the preference for the present is high (β is small), or if the long term and equilibrium nominal interest rate is weak (R), whereas the long term nominal growth rate ($y_i + \pi_i + \pi_i y_i$) is high.

Besides, if the previous increase of the public debt is higher, a higher Frisch labor supply elasticity (1/φ) limits the decrease of the real wage and of public investment, whereas public consumption increases less, and the decrease of labor supply and demand is accentuated. Finally, a higher equilibrium world real capital return (ρ), a situation of net capital exporter [$(K_{ij} - K_{ji})$ is higher], a higher long term capital taxation rate (t_i^k), or a weaker capital depreciation rate (δ) reduce the share of net exports in global economic activity in a given country (i), while they accentuate the decrease of these net exports.

Furthermore, even if the previous increase of the public debt has no consequence on private consumption or global public expenditure, it can modify the components of this public expenditure. Indeed, for a given ratio of long term public investment in comparison with consumption expenditure ($\frac{G_{inv.i}}{Gc,i}$), equations (31), (32), (A3.4), (A3.5) and (A3.12) imply:

$$\frac{\partial g_{inv.i,t}}{\partial\left(b_{i,t-1} - \pi_{i,t-1}\right)} = -\frac{\varphi(1-\nu)G_{c.i}(1-\beta)(1+R)}{(z_1 G_{c.i} - z_2 G_{inv.i})(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)} \tag{49}$$

$$\frac{\partial g_{c.i,t}}{\partial\left(b_{i,t-1} - \pi_{i,t-1}\right)} = \frac{\varphi(1-\nu)G_{inv.i}(1-\beta)(1+R)}{(z_1 G_{c.i} - z_2 G_{inv.i})(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)} \tag{50}$$

Therefore, a higher previous increase of the public debt decreases public investment expenditure [$\frac{\partial g_{inv.i,t}}{\partial(b_{i,t-1} - \pi_{i,t-1})} = -0.82$ with our basic calibration], while public consumption expenditure very slightly increases [$\frac{\partial g_{c.i,t}}{\partial(b_{i,t-1} - \pi_{i,t-1})} = 0.16$].

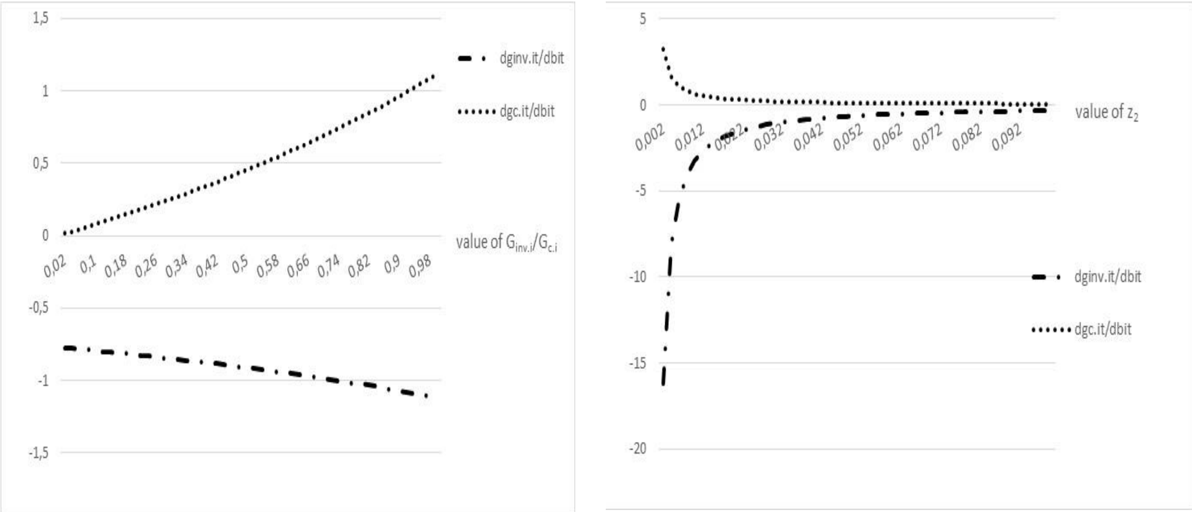


Figure 6: Effect of the previous increase of the public debt on the components of public expenditure, according to the long term ratio of public investment and consumption expenditure ($\frac{G_{inv.i}}{Gc,i}$) and to the productivity of public expenditure (z)

Besides, in the framework of a higher increase of the previous public debt, a higher capital share in the production function (ν) mostly increases the share of net exports in global demand, but it reduces the decrease of these net exports. It also slightly reduces the decrease of public investment expenditure and the increase of public consumption expenditure. A higher long term ratio of public investment in comparison with consumption expenditure ($\frac{G_{inv.i}}{Gc,i}$) increases the relative long term share of public investment; then, public investment decreases more, while public consumption on the contrary increases more, without any variation of global public expenditure. Finally, a higher productivity of public expenditure (z_2 is higher for a given z_1/z_2, or z_1/z_2 is higher for a given z_2) reduces the decrease of public investment expenditure, and it also reduces the increase of public consumption expenditure.

Therefore, if the increase of the public debt can be beneficial to current economic activity, it can also imply the risk to decrease the sustainable and long run economic growth for the future. Indeed, our model confirms the importance to take also into account the trajectory of the public debt, whether it is on an increasing or on a decreasing trend. For example, using data on a sample of 40 countries over the 1965-2010 period, Chudik *et al*. (2013) find significant negative long-run effects of public debt and inflation on growth, but only if the debt to GDP ratio is raised permanently. However, if this ratio increases (for example to smooth business cycle fluctuations) but is brought back afterwards to its 'normal' level, there would be no negative effect on long run economic growth. In the same way, using data for 118 developing, emerging and developed countries between 1960 and 2012, Eberhardt, and Presbitero (2015) find some support for a nonlinear relationship between debt and long-run growth across countries. However, they find no evidence for common debt thresholds within countries over time, and so, they underline the possibility of heterogeneities across countries, due to their own institutional framework, their own trajectories of public indebtedness and of economic shocks. In particular, the public debt would be less growth enhancing in the long run in countries with higher average debt burdens.

Finally, Ahlborn and Schweickert (2016) show that empirically, the institutional characteristics and the degree of fiscal uncertainty imply heterogeneities between the countries regarding the consequences of high public debt levels on their long-run economic growth. Continental (Core EU members) countries face more growth reducing public debt effects, especially for public debt levels beyond 75% of GDP. Indeed, these countries suffer from a lack of fiscal flexibility, because of a higher weight of the Welfare State in the pensions or redistribution systems ('subsidies for leisure', transfers). To the contrary, public debt apparently exerts neutral or even positive growth effects in Liberal (Anglo Saxon) countries, while for Nordic (Scandinavian) countries, giving a higher weight to 'transfers for work' (child and elderly care), a non-linear relationship exists, with negative debt effects around public debt values above 60% of GDP.

5 Conclusion

Our simple macroeconomic modelling shows that increasing the current public debt can sustain current economic activity, and all the more as the preference for the present of economic agents is high. It is mainly useful to increase net exports and public investment expenditure, whereas public consumption expenditure is left much more unchanged. The beneficial effect of the current public indebtedness on economic activity is also, obviously, accentuated if the long term and equilibrium nominal interest rate is weak whereas the long term nominal economic growth is high, and if the cost of the reimbursement of the public debt is reduced. Besides, the relative share of public investment in global public expenditure is higher if its productivity is weak. In the same way, on the labor market, a higher current public debt level could increase the capital stock, the real wage, and also more moderately labor demand and supply.

However, our modelling also underlines that there are many obstacles to the growth-enhancing effect of a higher public debt level on current economic activity. First, there would be a non-linear effect of this public debt. Indeed, if the public debt level is sufficiently low and considered as sustainable, an increase of this public debt can enhance economic growth. However, if the elasticity of the nominal interest rate to the increase of the public debt becomes high, if the nominal interest rate strongly increases because financial markets put into question the sustainability of this public debt, increasing the public indebtedness level can damage current economic activity. Besides, the previous increase of the public debt is also mostly harmful to global economic activity. Therefore, a previous increasing trend of the public debt can be damaging, and the trajectory of the public debt should be an important parameter to take into account. Indeed, if the public debt is on an increasing trend for many periods, it would be more damaging for current economic activity than if this public debt is on a decreasing trend, if it is under control and sustainable. So, in definitive, if increasing the current public debt could be beneficial to short run economic activity, it could also be harmful to long term economic growth, depending on its sustainability and on its long term trend.

Appendix 1: Main Components of Global Demand

Equations (12) and (15) imply the following long term production level:

$$Y_i = \frac{\rho}{\nu\left(1-t_i^k\right)}\left(K_{ii}+K_{ui}\right) \qquad (A1.1)$$

Using the equality between the demand and supply of capital ($K_{.,i}^{s} = K_{.,i}^{d}$), using equations (3), (9), (14), (15) and (A1.1), and with (B_i=0), we obtain the following long term share of private consumption in GDP:

$$\frac{C_i}{Y_i} = \frac{\left(1-t_i^l\right)(1-\nu)}{\left(1+t_i^c\right)} + \frac{\nu\left(1-t_i^k\right)\left(\rho-2\delta+\delta t_i^k\right)}{\left(1+t_i^c\right)\rho} + \frac{\nu\left[\left(1-t_u^k\right)\left(\rho-2\delta+\delta t_u^k\right)K_{iu}-\left(1-t_i^k\right)\left(\rho-2\delta+\delta t_i^k\right)K_{ui}\right]}{\left(1+t_i^c\right)\rho\left(K_{ii}+K_{ui}\right)} \qquad (A1.2)$$

Equations (14), (15), (20), (A1.1) and (A1.2) imply:

$$\frac{G_i}{Y_i} = \frac{\left[\nu\rho\left(t_i^k-t_i^l\right)-\nu\delta\left(2t_i^c+t_i^k\right)\left(1-t_i^k\right)+\left(t_i^c+t_i^l\right)\rho\right]}{\left(1+t_i^c\right)\rho} + \frac{\nu t_i^c\left[\left(1-t_u^k\right)\left(\rho-2\delta+\delta t_u^k\right)K_{iu}-\left(1-t_i^k\right)\left(\rho-2\delta+\delta t_i^k\right)K_{ui}\right]}{\left(1+t_i^c\right)\rho\left(K_{ii}+K_{ui}\right)} \qquad (A1.3)$$

So, if the relative shares of private and public consumption ($\frac{G_{inv.i}}{G_{C,i}}$) are defined, we have:

$$\frac{G_{c.i}}{Y_i} = \frac{1}{\left(1+\frac{G_{inv.i}}{G_{c.i}}\right)}\frac{G_i}{Y_i} \qquad \frac{G_{inv.i}}{Y_i} = \frac{1}{\left(1+\frac{G_{c.i}}{G_{inv.i}}\right)}\frac{G_i}{Y_i} \qquad (A1.4)$$

Equation (28) implies:

$$\frac{INV_{.i}}{Y_i} = \frac{\nu\delta\left(1-t_i^k\right)}{\rho} \qquad (A1.5)$$

Equations (25), (A1.2), (A1.3) and (A1.5) then imply:

$$\frac{EX_i}{Y_i} = \frac{\nu\delta(1-t_i^k)}{\rho} - \frac{\nu\left[\left(1-t_u^k\right)\left(\rho-2\delta+\delta t_u^k\right)K_{iu}-\left(1-t_i^k\right)\left(\rho-2\delta+\delta t_i^k\right)K_{ui}\right]}{\rho\left(K_{ii}+K_{ui}\right)} \qquad (A1.6)$$

Appendix 2: Optimal Output-Gap, Inflation, Capital stock and Public Debt

Equation (27) implies:

$$y_{i,t} = E_t\left(y_{i,t+1}\right) - \left[R_t - R_{t-1} - E_T\left(\pi_{i,t+1}\right) + \pi_{i,t} - \overline{R_{\iota,t}} + \overline{R_{\iota,t-1}}\right] \qquad (A2.1)$$

Equation (19) implies:

$$\left(\pi_{i,t}-\pi_{i,t-1}\right) = \beta\left[E_t\left(\pi_{i,t+1}\right) - E_t\left(\pi_{i,t}\right)\right] \qquad (A2.2)$$

Equations (24) and (A2.1) imply:

$$\widehat{b_{\iota,t}} = \frac{(1+\pi_i)(1+y_i)}{(1+R)}E_t\left(\widehat{b_{\iota,t+1}}\right) - \overline{R_{\iota,t}} + \overline{R_{\iota,t-1}} \qquad (A2.3)$$

Equations (10), (29) and (A2.1) imply:

$$k_{.i,t} = \frac{1}{(1-\delta)}E_t\left(k_{.i,t+1}\right) - \frac{\delta}{(1-\delta)}E_t\left(y_{i,t+1}\right) - \frac{\delta}{(1-\delta)}ln\left(\frac{1-t_{i,t}^k}{1-t_{i,t-1}^k}\right) + \frac{\delta}{(1-\delta)}\ln\left(\frac{\rho_t}{\rho_{t-1}}\right) + \frac{\delta}{(1-\delta)}\left[R_t - R_{t-1} - E_T\left(\pi_{i,t+1}\right) + \pi_{i,t} - \overline{R_{\iota,t}} + \overline{R_{\iota,t-1}}\right] \qquad (A2.4)$$

So, we have to solve the following system:

$$\begin{pmatrix} y_{i,t} \\ \pi_{i,t}-\pi_{i,t-1} \\ \widehat{b_{\iota,t}} \\ k_{.i,t} \end{pmatrix} = A \begin{pmatrix} E_T(y_{i,t+1}) \\ E_T(\pi_{i,t+1})-\pi_{i,t} \\ E_t(\widehat{b_{\iota,t+1}}) \\ E_t(k_{.i,t+1}) \end{pmatrix} + B \begin{pmatrix} \dfrac{R_{t-1}-R_t}{\overline{R_{\iota,t-1}}-\overline{R_{\iota,t}}} \\ ln\left(\dfrac{1-t_{i,t}^k}{1-t_{i,t-1}^k}\right) \\ \ln\left(\dfrac{\rho_t}{\rho_{t-1}}\right) \end{pmatrix} \qquad (A2.5)$$

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \dfrac{(1+\pi_i)(1+y_i)}{(1+R)} & 0 \\ -\dfrac{\delta}{(1-\delta)} & -\dfrac{\delta}{(1-\delta)} & 0 & \dfrac{1}{(1-\delta)} \end{pmatrix} \quad B \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\dfrac{\delta}{(1-\delta)} & \dfrac{\delta}{(1-\delta)} & -\dfrac{\delta}{(1-\delta)} & \dfrac{\delta}{(1-\delta)} \end{pmatrix}$$

With: $\lim_{n\to\infty} y_{i,n} = \lim_{n\to\infty}[\pi_{i,n}-\pi_{i,n-1}] = \lim_{n\to\infty} \widehat{b_{\iota,n}} = \lim_{n\to\infty} k_{.i,n} = 0$, this implies:

$$\begin{pmatrix} y_{i,t} \\ \pi_{i,t}-\pi_{i,t-1} \\ \widehat{b_{\iota,t}} \\ k_{.i,t} \end{pmatrix} = B\sum_{n=t}^{\infty} A^{n-t} \begin{pmatrix} \dfrac{R_{n-1}-R_n}{\overline{R_{\iota,n-1}}-\overline{R_{\iota,n}}} \\ ln\left(\dfrac{1-t_{i,n}^k}{1-t_{i,n-1}^k}\right) \\ \ln\left(\dfrac{\rho_n}{\rho_{n-1}}\right) \end{pmatrix} \qquad (A2.6)$$

Therefore, $A^n = \begin{pmatrix} 1 & e_n & 0 & 0 \\ 0 & \beta^n & 0 & 0 \\ 0 & 0 & \frac{(1+\pi_i)^n(1+y_i)^n}{(1+R)^n} & 0 \\ f_n & g_n & 0 & \frac{1}{(1-\delta)^n} \end{pmatrix}$,

- $e_n = 1+\beta e_{n-1} = \sum_{k=0}^{n-1}\beta^k = \dfrac{(1-\beta^n)}{(1-\beta)}$
- $f_n = -\dfrac{\delta}{(1-\delta)} + \dfrac{f_{n-1}}{(1-\delta)} = -\dfrac{\delta}{(1-\delta)}\sum_{k=0}^{n-1}\dfrac{1}{(1-\delta)^k} = 1-\dfrac{1}{(1-\delta)^n}$
- $g_0 = 0$; $g_n = -\dfrac{\delta e_{n-1}}{(1-\delta)} - \dfrac{\delta\beta^{n-1}}{(1-\delta)} + \dfrac{g_{n-1}}{(1-\delta)} = -\dfrac{\delta(1-\beta^n)}{(1-\delta)(1-\beta)} + \dfrac{g_{n-1}}{(1-\delta)}$

$$g_n = -\frac{\delta}{(1-\delta)(1-\beta)}\sum_{k=1}^{n}\frac{(1-\beta^k)}{(1-\delta)^{n-k}} \qquad for\ n>0$$

$$g_n = -\frac{[(1-\beta)-(1-\beta+\beta\delta-\delta\beta^{n+1})(1-\delta)^n]}{(1-\delta)^n(1-\beta)(1-\beta+\beta\delta)}$$

Economic variables are then as follows:

$$y_{i,t} = \sum_{n=t}^{\infty}[(R_{n-1}-R_n)+(e_{n-t}-\beta^{n-t})(\overline{R_{\iota,n-1}}-\overline{R_{\iota,n}})] \qquad (A2.7)$$

$$\pi_{i,t} = \pi_{i,t-1} \qquad (A2.8)$$

$$\widehat{b_{\iota,t}} = \sum_{n=t}^{\infty}\beta^{n-t}(\overline{R_{\iota,n-1}}-\overline{R_{\iota,n}}) = \overline{R_{\iota,t-1}}-(1-\beta)\sum_{n=t}^{\infty}\beta^{n-t}\overline{R_{\iota,n}} \qquad (A2.9)$$

$$k_{.i,t} = \frac{\delta}{(1-\delta)}\sum_{n=t}^{\infty}[(f_{n-t}-1)(R_{n-1}-R_n)+(\beta^{n-t}-e_{n-t}+g_{n-t})(\overline{R_{\iota,n-1}}-\overline{R_{\iota,n}})$$

$$-\frac{(1+\pi_i)^{n-t}(1+y_i)^{n-t}}{(1+R)^{n-t}} ln\left(\frac{1-t_{i,n}^k}{1-t_{i,n-1}^k}\right)+\frac{1}{(1-\delta)^{n-t}}\ln\left(\frac{\rho_n}{\rho_{n-1}}\right)] \qquad (A2.10)$$

Appendix 3: Equilibrium Economic Variables

Equations (A2.7) and (A2.9) imply:

$$y_{i,t}=R_{t-1}-\overline{R_{\iota,t-1}}+\sum_{n=t}^{\infty}(e_{n-t+1}-\beta^{n-t+1}-e_{n-t}+\beta^{n-t})\,\overline{R_{\iota,n}}$$

$$=R_{t-1}-\overline{R_{\iota,t-1}}+(2-\beta)\sum_{n=t}^{\infty}\beta^{n-t}\,\overline{R_{\iota,n}}=R_{t-1}+\frac{1}{(1-\beta)}\overline{R_{\iota,t-1}}-\frac{(2-\beta)}{(1-\beta)}\widehat{b_{\iota,t}} \qquad (A3.1)$$

Using $[\widehat{b_{\iota,t}}=b_{i,t}-\pi_{i,t}-y_{i,t}]$, equation (A3.1) implies:

$$y_{i,t}=-(1-\beta)R_{t-1}-\overline{R_{\iota,t-1}}+(2-\beta)(b_{i,t}-\pi_{i,t}) \qquad (A3.2)$$

Besides, equations (24) and (A2.1) imply:

$$\widehat{b_{\iota,t}}=\frac{(1+R)}{(1+\pi_i)(1+y_i)}\left(\widehat{b_{\iota,t-1}}+\overline{R_{\iota,t-1}}-\overline{R_{\iota,t-2}}\right) \qquad (A3.3)$$

So, by combining equations (A3.2) and (A3.3), we can obtain:

$$\overline{R_{\iota,t-1}}=-\frac{(1+\pi_i)(1+y_i)(1-\beta)}{(R-y_i-\pi_i-\pi_i y_i)}\left(b_{i,t}-\pi_{i,t}-R_{t-1}\right)$$

$$+\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}\left(b_{i,t-1}-\pi_{i,t-1}-R_{t-2}\right) \qquad (A3.4)$$

Therefore, equations (A3.2) and (A3.4) imply the following economic growth:

$$y_{i,t}=\frac{[1-\beta+(2-\beta)R-y_i-\pi_i-\pi_i y_i]}{(R-y_i-\pi_i-\pi_i y_i)}(b_{i,t}-\pi_{i,t})$$

$$-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}\left(b_{i,t-1}-\pi_{i,t-1}\right)-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}(R_{t-1}-R_{t-2}) \qquad (A3.5)$$

Equation (A2.10) implies:

$$k_{.i,t}=-\frac{\delta}{(1-\delta)}R_{t-1}-\frac{\delta^2}{(1-\delta)^2}\sum_{n=t}^{\infty}\frac{1}{(1-\delta)^{n-t}}R_n+\frac{\delta}{(1-\delta)}\overline{R_{\iota,t-1}}$$

$$-\frac{\delta^2}{(1-\delta)^2(1-\beta+\beta\delta)}\sum_{n=t}^{\infty}\frac{1}{(1-\delta)^{n-t}}\overline{R_{\iota,n}}-\frac{\delta(1-\beta)(2-\beta+\beta\delta)}{(1-\delta)(1-\beta+\beta\delta)}\sum_{n=t}^{\infty}\beta^{n-t}\,\overline{R_{\iota,n}}$$

$$-\frac{\delta}{(1-\delta)}\sum_{n=t}^{\infty}\frac{(1+\pi_i)^{n-t}(1+y_i)^{n-t}}{(1+R)^{n-t}} ln\left(\frac{1-t_{i,n}^k}{1-t_{i,n-1}^k}\right)+\sum_{n=t}^{\infty}\frac{\delta}{(1-\delta)^{n-t+1}}\ln\left(\frac{\rho_n}{\rho_{n-1}}\right) \qquad (A3.6)$$

Using equation (A3.4), we can obtain a value of $(k_{.i,t})$ according to the current and previous public debt levels. However, equations (12) and (16) also imply:

$$k_{.i,t}=\ln\left(\frac{1-t_{i,t}^k}{1-t_{i,t-1}^k}\right)+\left[(2-\beta)+\frac{(1-\beta)(1+\pi_i)(1+y_i)}{(R-y_i-\pi_i-\pi_i y_i)}\right](b_{i,t}-\pi_{i,t})-\ln(\frac{\rho_t}{\rho_{t-1}})$$

$$-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}\left(b_{i,t-1}-\pi_{i,t-1}\right)-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}(R_{t-1}-R_{t-2}) \qquad (A3.7)$$

Equations (16), (14) and (A3.7), or (13) and (A3.5) imply:

$$\left(w_{i,t}-\pi_{i,t}\right)+l_{i,t}=y_{i,t}=\frac{[1-\beta+(2-\beta)R-y_i-\pi_i-\pi_i y_i]}{(R-y_i-\pi_i-\pi_i y_i)}(b_{i,t}-\pi_{i,t})$$

$$-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}\left(b_{i,t-1}-\pi_{i,t-1}\right)-\frac{(1-\beta)(1+R)}{(R-y_i-\pi_i-\pi_i y_i)}(R_{t-1}-R_{t-2}) \qquad (A3.8)$$

Equations (7), (30), (A3.4) and (A3.5) imply:

$$\varphi l_{i,t} - \left(w_{i,t} - \pi_{i,t}\right) = \ln\left(\frac{1 - t_{i,t}^{l}}{1 - t_{i,t-1}^{l}}\right) + (1 - \beta)R_{t-1} - (2 - \beta)(b_{i,t} - \pi_{i,t}) \qquad (A3.9)$$

Then, by combining equations (A3.8) and (A3.9), we can obtain:

$$l_{i,t} = \frac{(1-\beta)(1+y_i)(1+\pi_i)}{(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}(b_{i,t} - \pi_{i,t}) + \frac{1}{(1+\varphi)}\ln\left(\frac{1 - t_{i,t}^{l}}{1 - t_{i,t-1}^{l}}\right) + \frac{(1-\beta)}{(1+\varphi)}R_{t-1} - \frac{(1-\beta)(1+R)}{(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left(b_{i,t-1} - \pi_{i,t-1} + R_{t-1} - R_{t-2}\right) \qquad (A3.10)$$

$$\left(w_{i,t} - \pi_{i,t}\right) = -\frac{1}{(1+\varphi)}\ln\left(\frac{1 - t_{i,t}^{l}}{1 - t_{i,t-1}^{l}}\right) - \frac{(1-\beta)}{(1+\varphi)}R_{t-1} + (2-\beta)(b_{i,t} - \pi_{i,t}) + \frac{\varphi(1-\beta)(1+\pi_i)(1+y_i)}{(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}(b_{i,t} - \pi_{i,t}) - \frac{\varphi(1-\beta)(1+R)}{(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left(b_{i,t-1} - \pi_{i,t-1} + R_{t-1} - R_{t-2}\right) \qquad (A3.11)$$

Finally, equations (11'), (31), (32), (A3.4), (A3.5), (A3.7) and (A3.10) can give the level of the shock on public investment. Indeed, they imply:

$$\varepsilon_{i,t}^{g,inv} = -\frac{\nu G_{c.i}}{(z_1 G_{c.i} - z_2 G_{inv.i})}\ln\left(\frac{1 - t_{i,t}^{k}}{1 - t_{i,t-1}^{k}}\right) - \frac{G_{c.i}}{(z_1 G_{c.i} - z_2 G_{inv.i})}a_{i,t} - \frac{(1-\nu)G_{c.i}}{(1+\varphi)(z_1 G_{c.i} - z_2 G_{inv.i})}\ln\left(\frac{1 - t_{i,t}^{l}}{1 - t_{i,t-1}^{l}}\right) + \frac{\nu G_{c.i}}{(z_1 G_{c.i} - z_2 G_{inv.i})}\ln\left(\frac{\rho_t}{\rho_{t-1}}\right) + \frac{(2-\beta)(1-\nu-z_1-z_2)G_{c.i}}{(z_1 G_{c.i} - z_2 G_{inv.i})}(b_{i,t} - \pi_{i,t}) - \frac{(1-\beta)(1+\pi_i)(1+y_i)}{(R - y_i - \pi_i - \pi_i y_i)}(b_{i,t} - \pi_{i,t}) + \frac{\varphi(1-\nu)G_{c.i}(1-\beta)(1+\pi_i)(1+y_i)}{(z_1 G_{c.i} - z_2 G_{inv.i})(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left(b_{i,t} - \pi_{i,t}\right) - \frac{\varphi(1-\nu)G_{c.i}(1-\beta)(1+R)}{(z_1 G_{c.i} - z_2 G_{inv.i})(1+\varphi)(R - y_i - \pi_i - \pi_i y_i)}\left(b_{i,t-1} - \pi_{i,t-1} + R_{t-1} - R_{t-2}\right) + \frac{(1-\beta)(1+R)}{(R - y_i - \pi_i - \pi_i y_i)}\left(b_{i,t-1} - \pi_{i,t-1} + R_{t-1} - R_{t-2}\right) + \frac{z_2(G_{c.i} + G_{inv.i})(1-\beta)}{(z_1 G_{c.i} - z_2 G_{inv.i})}R_{t-1} - \frac{G_{c.i}(1-\nu)(1-\beta)}{(1+\varphi)(z_1 G_{c.i} - z_2 G_{inv.i})}R_{t-1} + \frac{z_2 G_i}{(G_{c.i} z_1 - z_2 G_{inv.i})}\ln\left(\frac{1 + t_{i,t}^{c}}{1 + t_{i,t-1}^{c}}\right) \qquad (A3.12)$$

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