

Equilibrium Unemployment and the Finance of Unemployment Benefits

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Abstract

This paper examines the impact of a tax scheme that considers a flat tax rate on employment income on equilibrium employment within a general equilibrium framework, incorporating Shapiro and Stiglitz's (1984) theory of wage rigidity. We consider taxation of wages, which are used to finance, under a government's balanced budget regime, unemployment benefits, and analyse the effects on labour market outcomes. Our results show that the introduction of taxation on wages leads to an upward shift of the no-shirking condition (NSC) curve and a new equilibrium point at a higher level of unemployment and higher wages. The results underscore the significance of tax scheme design in shaping labour market equilibrium. These results may have a role in policy decisions aimed at promoting employment and economic growth and offer some insight for empirical studies on the estimation of the level of equilibrium unemployment.

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1. Introduction

The Shapiro and Stiglitz (1984) model is a well-known framework for analysing the labour market and the impact of unemployment on wages. In particular, the authors argue that unemployment can serve as a disciplinary device, motivating workers to exert effort and preventing shirking. They introduce the concept of "efficiency wages", where firms pay workers a wage above the market-clearing level to induce them to work efficiently, as the threat of unemployment provides a strong incentive to perform.

The efficiency wage literature has evolved significantly since the introduction of the no-shirking model by Shapiro and Stiglitz (1984), who demonstrated that firms may pay wages above the competitive equilibrium to incentivize effort and prevent shirking. Subsequent studies (e.g., Akerlof & Yellen, 1986; Romer, 1996) have explored the implications of efficiency wages for unemployment and wage rigidity. Pissarides (1998) and others have examined the role of labour market frictions, showing that even modest institutional changes can lead to large welfare implications. Recent contributions have expanded the model to include government policy interactions. For instance, Rasmussen (2002) and Røed, K., & Strøm, S. (2002) analyse how progressive taxation can alter wage structures and employment levels. Our work is close to Pisauo (1991), who analyses the effects of labour taxation on wage and employment in a moral hazard-efficiency wage model.

This paper extends the Shapiro and Stiglitz (1984) framework, as formally presented in Romer (1996), to examine the effects of alternative taxation policies on equilibrium unemployment, with a focus on assessing the impact of various fiscal instruments that governments can employ to implement a balanced budget policy, particularly in relation to unemployment benefits. In other words, the government raises revenues through taxation to pay the unemployment benefits under a balanced budget regime, which is our main differentiation to the existing literature.

2. The model

We study the direct taxation of employee wages. The government seeks to equalize this revenue with the total unemployment benefits it pays out. For the purpose of this analysis, we will focus solely on balancing the government's budget with respect to this specific expenditure and assume that it is the only expenditure to consider.

We consider an economy in which there is a large number of identical workers \bar{L} , and a large number of identical firms N . Workers want to maximise their expected utility and firms to maximise their expected profits.

The lifelong utility of the representative worker is:

$$U = \int_{t=0}^{\infty} e^{-\rho t} u(t) dt, \quad \rho > 0 \quad (1)$$

where $u(t)$ is the instantaneous utility in time t , and ρ is the discount rate.

The instantaneous utility is:

$$u(t) = \begin{cases} w(t) - e(t) - w(t)\tau \\ w(t) - w(t)\tau \\ \bar{w} \end{cases} \quad (2)$$

where w it is the wage and e the effort made by the employee as in Shapiro and Stiglitz (1984) and \bar{w} is the level of unemployment benefit. We consider that the effort is not a constant variable, and that there are two levels of effort e which are $e = 0$ and $e = \bar{e}$, $0 < \tau < 1$ is the tax rate, $w(t) - e(t) - w(t)\tau$ is the

instantaneous utility of the worker who makes an effort, $w(t) - w(t)\tau$ the instantaneous utility of the worker who does not make any effort, and \bar{w} the instantaneous utility of the unemployed. This allows the worker to be in one of the three situations: be employed and make an effort, which we denote with E , be employed and not to make an effort that we denote with S , and be unemployed which is denoted by U .

In line with Shapiro and Stiglitz (1984) and Romer (1996), the assumptions concerning the transition of employees between the three situations mentioned above are a crucial part of the model.

We firstly assume that there is an exogenous rate at which the employment of worker changes states, either due to their own choice or due to the firm's decisions. If a worker starts working in a particular job at some point in time t_0 and this worker exerts effort, the possibility that the worker will be in the same position in the future t is:

$$P(t) = e^{-bt(t-t_0)} \quad b > 0 \quad (3)$$

Equation (3) implies that $P(t + \tau) / P(t) = e^{-b\tau}$, and is therefore independent from time t . In other words, if a worker is employed the probability of remaining employed at time τ time is $e^{-b\tau}$ regardless of how long the employee was already employed. This lack of temporal dependency simplifies the analysis because we do not need to monitor how long workers retain their position.

For our second crucial assumption and following the shirking model of Shapiro and Stiglitz, we consider that firms monitor employees for shirking, and this monitor technology is also independent to time t . In particular, detection of shirking occurs with probability q per unit of time, which is exogenous. Employees that are found to shirk, are dismissed. Thus, the probability that a worker who chooses not to do the required effort (\bar{e}) is still employed after time τ is $e^{-q\tau}$ (the probability of the worker not being caught shirking) times $e^{-b\tau}$ (the likelihood that his or her work will not have ceased for other external reasons).

Following Shapiro and Stiglitz, and Romer, our third assumption is that at any given time total unemployment is α , which is exogenous for each individual worker, but is determined endogenously. This rate α is determined by the firms' behaviour and the percentage of recruitment they aim to achieve from the overall pool of unemployed individuals. The rate of recruitment is also influenced by the pace at which employment arrangements with certain workers end, either voluntarily or involuntarily, as well as the frequency at which underperforming employees are identified and terminated due to a lack of necessary effort. Because of the assumed homogeneity, the likelihood of finding new employment is not affected by the manner in which someone is or became unemployed, or how long they are unemployed. The firms aim to maximising its profits by choosing wages capable of preventing shirking.

The profits of the representative firm at the time t are:

$$\pi(t) = F(\bar{e}L(t)) - w(t)L(t) \quad F' > 0, \quad F'' < 0, \quad (4)$$

where L is the number of employees. At every moment, the firm chooses L and w to maximise the instantaneous profit flow.

Our final hypothesis which again is in-line with Shapiro and Stiglitz, and Romer, is that $\bar{e}F'(\bar{e}\bar{L}/N) > \bar{e}$ or $F'(\bar{e}\bar{L}/N) > 1$, which implies that if each firm hires $1/N$ of the workforce, the marginal output of the work outweighs the costs for exerting effort. Thus, if monitoring were perfect there would be full employment.

Following Romer's analysis, we consider that V_i is the 'value' of the state i , where $i = E, S, U$. In other words, V_i is the expected value of the discounted lifetime utility of the worker at state i . The transition from one state to another is a Poisson process and V_i does not depend on how long the worker is in state i or their previous employment history. We focus on the stable state, so V_i is stable in time. We focus on a short period of time and use V_i to see what happens at the end this period. We consider a worker who is

employed and makes no effort ($e=0$). We explore short time intervals Δt , where a worker who loses his or her job cannot start looking for new employment before the next time interval. We then consider the values of employment $V_E(\Delta t)$ and unemployment $V_U(\Delta t)$ at the beginning of a short time period. Further below we will explore what happens when Δt is approaching 0.

By doing so, the restriction that a worker in Δt cannot seek new employment after losing their current job is no longer relevant. This $V_E(\Delta t)$ will tend to V_E . Here, we diverge from the model of Shapiro and Stiglitz due to the assumption we made in equation (2), i.e., introduction of taxation on employees' wages. Thus, if an employee is employed (and makes an effort) that and receives wage w , then this is $V_E(\Delta t)$ given by the equation:

$$V_E(\Delta t) = \int_{t=0}^{\Delta t} e^{-bt} e^{-\rho t} (w - \bar{e} - w\tau) dt + e^{-\rho \Delta t} [e^{-b\Delta t} V_E(\Delta t) + (1 - e^{-b\Delta t}) V_U(\Delta t)] \quad (5)$$

The first term of equation (5) is the utility during the time interval $(0, \Delta t)$. The probability that the worker still be employed at time t is e^{-bt} . If the employee is employed and makes an effort, then their instantaneous utility is $w - \bar{e} - w\tau$. Discounting it to time 0 results in the contribution of this term to life-long usefulness which is $e^{-\rho t} (w - \bar{e} - w\tau)$.

The second term of equation (5) is the utility after the time interval Δt . In time Δt the worker has a probability to be employed of $e^{-b\Delta t}$ and a probability of being unemployed of $1 - e^{-b\Delta t}$. Combining probabilities with corresponding utilities results in the second term.

We then calculate its integral of (5) and find:

$$V_E(\Delta t) = \frac{1}{\rho + b} (1 - e^{-(b+\rho)\Delta t}) (w - \bar{e} - w\tau) + e^{-\rho \Delta t} [e^{-b\Delta t} V_E(\Delta t) + (1 - e^{-b\Delta t}) V_U(\Delta t)] \quad (6)$$

The next step is to solve (6) $V_E(\Delta t)$ which gives us:

$$V_E(\Delta t) = \frac{1}{\rho + b} (w - \bar{e} - w\tau) + \frac{1}{1 - e^{-(b+\rho)\Delta t}} e^{-\rho \Delta t} (1 - e^{-b\Delta t}) V_U(\Delta t) \quad (7)$$

As previously mentioned, this V_E is equal to $V_E(\Delta t)$ as Δt tends to zero. To find this limit, we apply the L'Hopital rule on equation (7) and have:

$$V_E = \frac{1}{\rho + b} [(w - \bar{e} - w\tau) + b V_U] \quad (8)$$

which can also be written as:

$$\rho V_E = (w - \bar{e} - w\tau) - b(V_E - V_U) \quad (9)$$

Similarly, if a worker is employed, does not make an effort, and receives wage w , then $V_S(\Delta t)$ is given by the equation:

$$V_S(\Delta t) = \int_{t=0}^{\Delta t} e^{-bt} e^{-\rho t} e^{-qt} (w - w\tau) dt + e^{-\rho \Delta t} [e^{-(b+q)\Delta t} V_E(\Delta t) + (1 - e^{-(b+q)\Delta t}) V_U(\Delta t)] \quad (10)$$

Following the same steps as before yields:

$$\rho V_S = (w - w\tau) - (b + q)(V_S - V_U) \quad (11)$$

And finally, in-line with Shapiro and Stiglitz, and Romer, the same procedure for an unemployed person gives us:

$$\rho V_U = \bar{w} + \alpha(V_E - V_U) \quad (12)$$

where \bar{w} is the unemployment benefit paid by the government.

The firm must choose a wage such to prevent shirking ($V_E \geq V_S$). As the effort cannot be greater than \bar{e} the firm sets wages so that:

$$V_E = V_S \quad (13)$$

From equations (9), (11) and (13) we get to the following relation:

$$V_E - V_U = \frac{\bar{e}}{q} \quad (14)$$

Equation (14) means that companies set wages at a level that workers strictly prefer employment to unemployment.

From equation (9) we have:

$$\begin{aligned} \rho V_E &= (w - \bar{e} - w\tau) - b(V_E - V_U) \Rightarrow w - w\tau = \bar{e} + \rho V_E + b(V_E - V_U) = \\ &= \bar{e} + \rho V_E - \rho V_U + \rho V_U + b(V_E - V_U) \Rightarrow \\ w(1 - \tau) &= \bar{e} + \rho V_U + (\rho + b)(V_E - V_U) \end{aligned} \quad (15)$$

and combining (15) with (12) gives us:

$$w(1 - \tau) = \bar{e} + \bar{w} + (\alpha + \rho + b)(V_E - V_U) \quad (16)$$

which this in turn gives us (considering (14)):

$$w = \frac{1}{(1 - \tau)} \left[\bar{e} + \bar{w} + (\alpha + \rho + b) \frac{\bar{e}}{q} \right] \quad (17)$$

Equation (17) shows that the wage increases in function with effort \bar{e} , is positively affected by the level of unemployment benefits \bar{w} , the ease in which workers can find new employment α , the discount rate ρ , the exogenous pace at which employment contracts cease b , and is decreasing in function with the probability that the firm can identify those who do not shirk. Finally, when the tax rate τ increases then, *ceteris paribus*, the term $\frac{1}{(1 - \tau)}$ also increases, and thus increases the effective wage that the company must pay.

Following Shapiro and Stiglitz, and Romer, we will express (17) in terms of employment per company L (rather than in relation to the rate at which unemployed people enter employment α). To replace α ,

we use the fact that once the economy is in a stable state equilibrium. In a steady-state labour market, the number of people entering employment (inflow) must be equal to the number of people leaving employment (outflow). The number of employees leaving employment at any given unit of time is N (the number of enterprises) multiplied by L (the number of employees per enterprise) multiplied by b (the exogenous percentage at which employment contracts cease). The number of unemployed findings work in the unit of time is $\bar{L} - NL$ multiplied by α . By equalising these two quantities we have:

$$\alpha = \frac{NLb}{\bar{L} - NL} \quad (18)$$

Which can be written:

$$\begin{aligned} \alpha &= \frac{NLb}{\bar{L} - NL} \Rightarrow \alpha + b = \frac{NLb}{\bar{L} - NL} + b \Rightarrow \alpha + b = \frac{NLb + \bar{L}b - NLb}{\bar{L} - NL} \\ &\Rightarrow \alpha + b = \frac{\bar{L}b}{\bar{L} - NL} \end{aligned} \quad (19)$$

replacing (19) into (17) results in:

$$w = \frac{1}{(1-\tau)} \left[\bar{e} + \bar{w} + (\rho + b \frac{\bar{L}}{\bar{L} - NL}) \frac{\bar{e}}{q} \right] \quad (20)$$

Equation (20) is the no-shirking condition (NSC). The NSC is a function of the level of unemployment, the level of unemployment benefits, and the tax rate the wage the firm will have to pay to stimulate an effort from its employees.

Regarding the treatment of firms, our approach does not differ from that of Shapiro and Stiglitz, and Romer. Firms hire workers up to the point where the marginal output of the work equals the wage. This condition that results from the maximising behaviour of firms, and considering equation (4), is:

$$\bar{e}F'(\bar{e}L) = w \quad \text{with } F' > 0, F'' < 0, \quad (21)$$

which is the labour demand curve.

Finally, the government wants to finance unemployment benefits through taxes collected from workers (wage taxes). At any given moment, the government runs a balanced budget and the revenue of the government is N (the number of enterprises) multiplied L (the number of employees in each enterprise) by w (the wage received by each employee) by τ (the tax rate). This revenue must be equal to the expense \bar{w} (the unemployment benefit received by each worker) multiplied by $(\bar{L} - NL)$ (the number of unemployed persons).

Thus:

$$NLw\tau = \bar{w}(\bar{L} - NL) \quad (22)$$

Our model results in the following system of equations:

$$\begin{cases} w = \frac{1}{(1-\tau)} [\bar{e} + \bar{w} + (\rho + b \frac{\bar{L}}{\bar{L} - NL}) \frac{\bar{e}}{q}] \\ \bar{e}F'(\bar{e}L) = w \\ NLw\tau = \bar{w}(\bar{L} - NL) \end{cases} \quad F' > 0, F'' < 0, \text{ (p 4.1),}$$

The unknown variables in our system are L, w, τ and \bar{w} , while $N, \bar{L}, \bar{e}, \rho, b$ and q are parameters. The underlying concept behind these equations is that the firm sets a wage level that it has no incentive of reducing, as it understands that paying a lower wage would result in employees not putting in the required level of effort. However, this approach ultimately leads to an equilibrium characterized by involuntary unemployment. As part of its social policy, the government provides unemployment benefits to the unemployed to ease their economic burden until they return to employment. However, this unemployment benefit has an unintended consequence: it reduces the motivation of employed to exert the required effort, and to start shirking, as they know that if they are caught and dismissed they will receive an unemployment benefit.

The firm understands this and increases wages even further to increase the cost of the worker's loss of work, as the wage is higher than the unemployment benefit, but it also leads to higher unemployment which also pushes wages downwards (see eq. (20)).

By taxing employees to pay unemployment benefits again, the government reduces the employee's cost of losing work as the actual wage he receives is reduced by $(1 - \tau)$. This forces the company to increase wages more, leading to higher unemployment, leading to lower wages. The system we reached is a 3x3 non-linear system. To simplify it, it will be assumed that the production function is Cobb-Douglas in its simplest form:

$$F(\bar{e}L) = (\bar{e}L)^a \quad (23)$$

where elasticity $0 < a < 1$, and we see conditions $F' > 0, F'' < 0$, are satisfied, the system can be described as follows:

$$\begin{cases} w = \frac{1}{(1-\tau)} [\bar{e} + \bar{w} + (\rho + b \frac{\bar{L}}{\bar{L} - NL}) \frac{\bar{e}}{q}] \\ \bar{e}a(\bar{e}L)^{a-1} = w \\ \tau = \frac{\bar{w}(\bar{L} - NL)}{NLw} \end{cases} \quad (23.1)$$

and by replacing the tax rate in the first equation and solving for w will give us the new NSC equation, which, in conjunction with the second equation which is a function of labour demand, will determine the equilibrium. The system is therefore rewritten as:

$$\begin{cases} w = \bar{e} + \bar{w} + (\rho + b \frac{\bar{L}}{\bar{L} - NL}) \frac{\bar{e}}{q} + \frac{\bar{w}(\bar{L} - NL)}{NL} \\ \bar{e}a(\bar{e}L)^{a-1} = w \\ \tau = \frac{\bar{w}(\bar{L} - NL)}{NLw} \end{cases} \quad (23.2)$$

The new NSC is similar to that of Shapiro and Stiglitz, except for the last term, which is also naturally positive. Intuitively, we would expect an upward shift of the NSC curve, compared to that in the Shapiro and Stiglitz.

We explore its properties to understand its form. The first derivative of the NSC is:

$$\frac{\partial w}{\partial L} = \frac{b\bar{e}\bar{L}N}{(\bar{L} - LN)^2 q} - \frac{\bar{w}}{L} - \frac{(\bar{L} - LN)\bar{w}}{L^2 N} \quad (24)$$

The second derivative is:

$$\frac{\partial^2 w}{\partial L^2} = \frac{2b\bar{e}\bar{L}N^2}{(\bar{L} - LN)^2 q} + \frac{2\bar{w}}{L^2} + \frac{2(\bar{L} - LN)\bar{w}}{L^3 N} \quad (25)$$

which is always positive and therefore the NSC is concave. The first derivative is set to zero for a positive value L and this point is therefore the minimal. This L is:

$$L = \frac{\bar{L}\sqrt{q\bar{w}}}{N(\sqrt{b\bar{e}} + \sqrt{q\bar{w}})} \quad (26)$$

Even if we take the NSC limit for $NL \rightarrow \bar{L}$ and $NL \rightarrow 0$, becomes infinite, i.e., exhibits asymptotes. For the labour demand curve we have:

$$\frac{\partial w}{\partial L} = \bar{e}^2 a(a-1)(\bar{e}L)^{a-2} < 0 \text{ and } \frac{\partial^2 w}{\partial L^2} = \bar{e}^3 a(a-1)(a-2)(\bar{e}L)^{a-3} > 0 \quad (27)$$

the curve is therefore decreasing in L and concave.

Because of the above, we expect a behaviour of the system as shown in Figure 1.

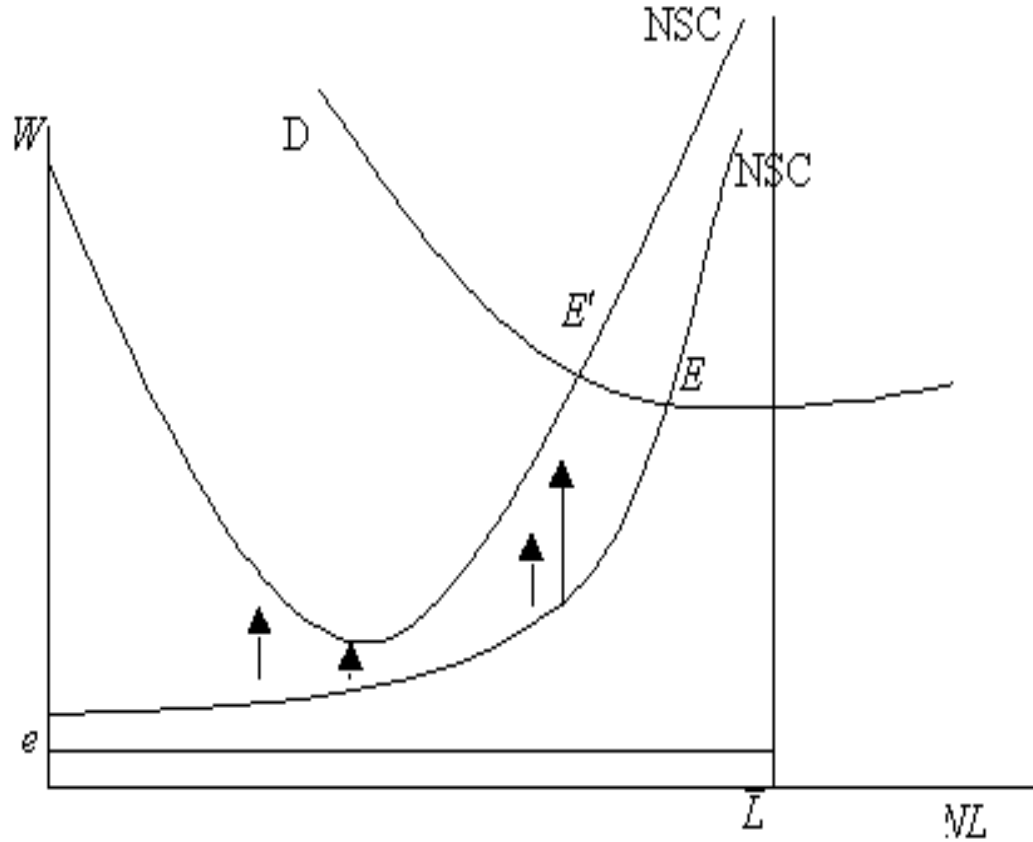


Figure 1: The change of the NSC

Note: This figure presents graphically the equilibrium of unemployment in the Shapiro and Stiglitz's (1984) model (point E), and new NSC resulting into a higher unemployment rate (point 'E').

In other words, the behaviour of the NSC from strictly increasing, is initially decreasing to the point

$L = \frac{\bar{L}\sqrt{q\bar{w}}}{N(\sqrt{b\bar{e}} + \sqrt{q\bar{w}})}$ and then increasing until it approaches the vertical asymptote \bar{L} . Moving upwards

creates a new equilibrium which is to left and higher than E (from E to E'). This means that the new equilibrium translates into higher wages and lower employment.

Therefore, taxing the employee's wage reduces the actual wage below the effective wage, thus, workers do not have an incentive to exert the effort.

Firms perceive this and thus create a new effective wage threshold higher than the previous one to incentivise effective work. As a result, higher wages reduce labour demand and increase involuntary unemployment. However, the model, also operating as an 'employee discipline mechanism' will give the signal of increased unemployment to workers and businesses, so the NSC will move slightly less and balance as shown in Figure 1.

3. Conclusion

In this paper, we built on the literature of efficiency wages theory to also consider the finance of unemployment benefits through taxation of employment income, under a balanced budget regime. To do so we utilized a simplified version of the Shapiro and Stiglitz's (1984) well known shirking model. The main finding is that when the government pays unemployment benefits that are financed by a direct flat tax on the wage rate and is running a balanced budget, it causes an upward shift in the non-shirking condition that leads to higher efficient wage and a higher unemployment rate. It is natural to consider this result with caution since this is simplified model that is abstract from real economies, but we consider it a useful exercise to highlight the potential role of fiscal policy in the determination of equilibrium unemployment. This finding may also be useful for empirical work. For example, Blanchard (2018) and Blanchard and Katz (1997) argue that we have a limited quantitative understanding of the determinants of the natural rate of unemployment, especially across different countries, and the impact of different fiscal policies may be an important feature in future econometric models.

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