

# Hedge ratio estimation: A note on the Bitcoin future contract

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## Abstract

This paper investigates the hedging effectiveness of Bitcoin (BTC) future contract using daily settlement prices for the period of 1 January 2018 until 26 March 2021. Standard OLS regressions, Error Correction Model (ECM), as well as GARCH and EGARCH models are used to estimate the optimal hedge ratio which is necessary for trading and risk management. The findings indicate that the time varying hedge ratios, if estimated through the Error Correction Model (ECM), are more efficient than the fixed hedge ratios in terms of risk minimization.

**JEL classification numbers:** G11, G13

**Keywords:** Optimal hedge ratio, hedging models, bitcoin, futures market.

## 1 Introduction

Financial markets have been highly volatile and highly complex in recent decades. As a result, the determination of optimal hedge ratios has emerged as the main subject of discussion for the academic community mainly for risk management purposes. It is also in the center of attention by the majority of financial institutions, investors and businesses, here the focus is tilted towards trading and effective asset management. The main issue which is key to hedge ratios relates to the number of futures that the investor can hold for each underlying unit in order to protect its portfolio against any undesirable market movements. The main objective of the paper is the direct comparison, through a trading strategy process, of the forecasting ability of several econometric approaches that account for the hedging effectiveness. Hedging through trading futures is a common process which is used to control or even reduce the risk of adverse price movements. One of the most important theoretical issues in Risk Management is finding the optimal risk hedge ratio. The question is to find the optimal method so that investors are protected against possible undesirable market movements at the lowest possible cost.

In the literature, the first model was developed by Johnson (1960) and then improved by Ederington (1979) who minimized the variation of the total portfolio using the Ordinary Least Square Method (OLS). However, Park and Berra (1987) and Herbst (1989) questioned its importance since the approaches above do not take into account the heteroskedasticity that exists between the underlying and the futures contract. Criticism has also been extended to the fact that the hedge ratios of an OLS model are static, that is, they do not take into account the variability in time-bound variance as well as the alteration of other parameters such as curvature and symmetry.

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Lypny and Powalla (1998) examined the hedging effectiveness of the German stock index DAX futures and showed that the application of a dynamic hedging strategy based on a GARCH (1,1) process is economically and statistically the most effective model.

Butterworth and Holmes (2001) investigated the hedging effectiveness of the FTSE-Mid 250 stock index futures contract using actual diversified portfolios in the form of Investment Trust Companies (ITCs). Using an alternative econometric technique (Least Trimmed Squares Approach) to estimate hedge ratios, their results showed that this contract is superior to the FTSE-100 index futures contract when hedging cash portfolios which mirrors the Mid250 and the FT Investment Trust (FTIT) indices.

Chen et al. (2001) extended the GSV hedge ratio to the Mean-GSV hedge ratio. Switzer and Khoury (2007) showed that hedging performance is improved when asymmetry of extreme volatilities is considered. Norden (2006) found the hedging efficiency is significantly increased after the futures split.

Kenourgios et al. (2008) examined the hedging effectiveness of the USA stock index S&P futures and showed that in terms of risk reduction ECM is the appropriate method for estimating optimal hedge ratios as it provides better results than the conventional OLS method, the ECM with GARCH errors, the GARCH model and the EGARCH (1,1) model.

Lee et al (2009) examined four static and one dynamic hedging model by using the data from Taiwan, United States, Japan, Hong Kong, Singapore and Korean to find the optimal hedge ratios. They found that although optimal hedge ratios differ in each market, the use of stock index future does provide an effective instrument for hedging irrespectively of the strategy or the time horizon employed.

Wang and Hsu (2010) empirically studied the hedge ratio stability of the Japan, Hong Kong and Korean index futures contracts during the Asian financial crisis and post-crisis. They concluded that time varying hedge ratios, if estimated through the Autoregressive Distributed Lags (ARDL), are more efficient than the fixed hedge ratios in terms risk minimization. Sahb and Pandey (2011) used daily data for the S&P CNX Nifty futures to estimate the effective hedge ratio and its hedging effectiveness of three models. They also concluded that Nifty futures contract provide an effective contract for hedging purposes.

Jian Zhou (2016) extended the literature in hedging performance by examining what hedge-ratio estimation method yields the most effective hedging performance of Real Estate Investment Trusts futures. Koulis et al. (2018) investigated the hedging effectiveness of the International Index Futures Markets. Their findings again indicated that the time varying hedge ratios, if estimated through the ARDL model, are more efficient than the fixed hedge ratios in terms of minimizing the risk.

Beneki et al. (2019) set out to test the hypothesis whether volatility spillovers and hedging abilities exist between Bitcoin and Ethereum by a multivariate BEKK-GARCH methodology. The findings revealed significant swaps in the time-varying correlation and a delayed positive response of Bitcoin volatility on a positive volatility shock on Ethereum returns.

Pal and Mitra (2019) computed optimal hedge ratios between bitcoin and other financial assets by using conditional volatility. The results showed that Gold is found to provide a better hedge against bitcoin.

Sebastião and Godinho, P. (2020) investigated the hedging properties of CBOE Bitcoin futures during the initial months of trading. The results pointed out that bitcoin futures are effective hedging instruments not only for bitcoin, but also for other major cryptocurrencies.

Deng et al. (2020) formulated an optimal hedging problem of Bitcoin inverse futures under the minimum-variance framework. The results showed that the optimal hedging strategy achieves superior effectiveness in reducing risk and outperforms the naïve hedge strategy in all scenarios.

## **2 Methodology**

This paper aims to determine the appropriate model when estimating optimal hedge ratios. The alternative models employed are as follows.

### 2.1 Model 1: the conventional regression model

This model is just a linear regression of change in spot prices on changes in futures prices. Let  $S_t$  and  $F_t$  be logged spot and futures prices, respectively, hedge ratio can be estimated as follows:

$$\Delta S_t = \alpha_0 + \beta \Delta F_t + u_t \quad (1)$$

where  $u_t$  is the error from the OLS estimation,  $\Delta S_t$  and  $\Delta F_t$  represent spot and futures price changes and the slope coefficient  $\beta$  is the optimal hedge ratio conventionally denoted as  $h^*$ .

### 2.2 Model 2: the error correction model

Engle and Granger (1987) stated that if sets of series are cointegrated, then there exists a valid error correction representation of the data. Thus, if  $S_t$  represents the index spot price series and  $F_t$  the index of futures price series and if both series are  $I(1)$ -integration of order 1-, there exists an error correction representation of the following form:

$$\Delta S_t = \alpha u_{t-1} + \beta \Delta F_t + \sum_{k=1}^m \theta \Delta F_{t-k} + \sum_{j=1}^n \varphi \Delta S_{t-j} + e_t \quad (2)$$

where  $u_{t-1} = S_{t-1} - [a_0 + a_1 F_{t-1}]$  is the Error Correction Term and has no moving average part; the systematic dynamics is kept as simple as possible and enough lagged variables are included to ensure that  $e_t$  is a white noise process and the coefficient  $\beta$  is the optimal hedge ratio.

### 2.3 Model 3: the GARCH model

A useful generalization of ARCH models introduced by Bollerslev (1986) is the GARCH (1,1) model that parameterizes volatility as a function of unexpected information shocks to the market. The equation for GARCH (1,1) is as follows:

$$\sigma_t^2 = a_0 + a_1 e_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3)$$

The equation specified above is a function of three terms: the mean  $a_0$ , news about volatility from the previous period measured as the lag of the squared residual from the mean equation  $e_{t-1}^2$  (the ARCH term), and last period's forecast variance  $\sigma_{t-1}^2$  (the GARCH term). The more general GARCH ( $p, q$ ) calculates  $\sigma_t^2$  from the most recent  $p$  observations on  $e^2$  and the most recent  $q$  estimates of the variance rate.

### 2.4 Model 4: the EGARCH model

The EGARCH model is given by

$$\log \sigma_t^2 = \bar{\omega} + \beta \log(\sigma_{t-1}^2) + \gamma \left( \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| \quad (4)$$

where  $\bar{\omega}$ ,  $\alpha$ ,  $\beta$  and  $\gamma$  are constant parameters (Nelson, 1991). The left-hand side is that of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic and those forecasts of the conditional variance are guaranteed to be non-negative.

### 3 Data and empirical results

The data employed in this study comprise 1007 daily observations on the BTC spot and BTC futures contract (January 2018–March 2021). Closing prices for spot and futures were obtained from [www.investing.com](http://www.investing.com) which is shown in Fig. 1 & 2. We notice that since the middle of 2020 there has been an exponential rise of both series. For both futures and spot price series daily returns were calculated as  $\log(p_t/p_{t-1})$  and the related summary statistics exist in Table 1. It shows that the standard deviation of the return for the BTC is approximately close to that for the BTC future, meaning that the return risks are similar. The Jarque-Bera statistics in Table 1 reveals that both series are not normally distributed. Additionally, the correlation coefficient indicates that both series are highly correlated.

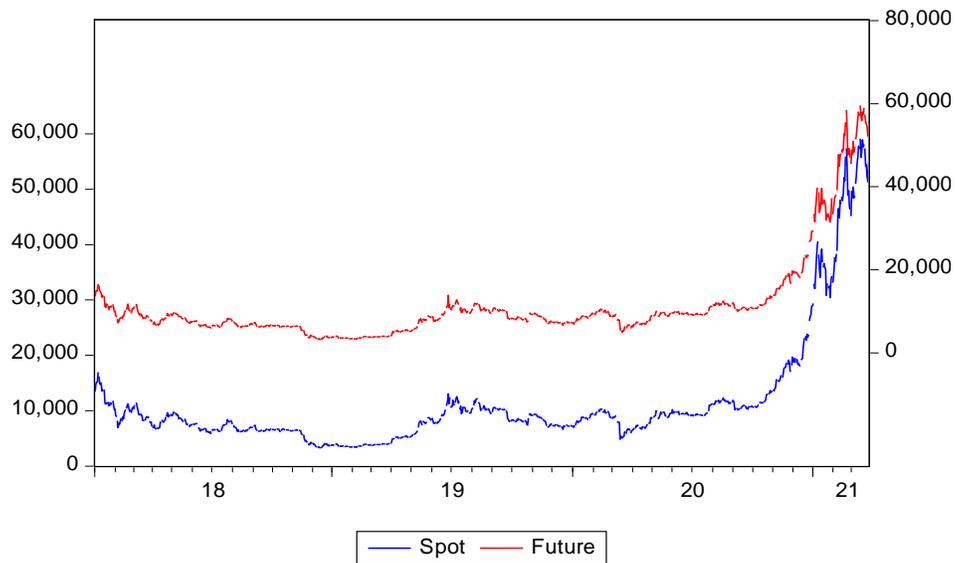


Figure 1: Series for BTC Index and BTC Futures prices

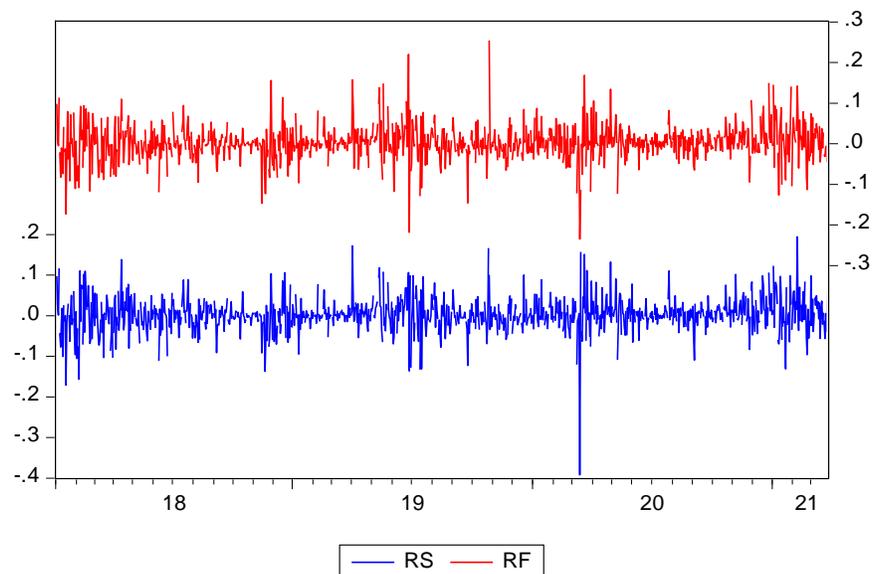


Figure 2: Series for BTC Index and BTC Futures returns

Table 1: Summary Statistics

	<b>Spot</b>	<b>Future</b>
Mean	0.002357	0.002335
Median	0.001657	0.000809
Maximum	0.194137	0.254477
Minimum	-0.391816	-0.234882
Std. Dev.	0.043082	0.043979
Skewness	-0.603254	0.043744
Kurtosis	11.62063	7.253324
Jarque-Bera	3172.904	757.8712
Probability	0.000000	0.000000
Sum	2.368298	2.346635
Sum Sq. Dev.	1.863473	1.941857
Observations	1005	1005
Correlation	0.878923	

### Unit root test

Tests for the presence of unit root are performed by conducting the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests under the assumption that there is no linear trend in the data generation process. However, after plotting the data we have identified that both our series appear to be trended. Therefore, the tests were performed using a linear time trend and an intercept. The ADF (four lags) and PP (five lags) test statistics indicate that none of the level series are stationary processes; while for the differenced series the hypothesis of a unit root is rejected at 5% level, suggesting that the differenced series are stationary processes. The test results are reported in Table 2.

Table 2: Results of the Unit-Root Tests on Spot and Futures Prices

	<b>ADF</b>	<b>PP</b>
Spot	-31.78380*	-31.90240*
Future	-9.721325*	-33.00261*

Note: The null hypothesis is that series has a unit root.

\*Denotes that the test statistics are significantly different from zero at the 5% level.

Table 3 presents the results of the estimate hedge ratio based on four models described above. We note that the hedge ratio is lower than the unit and the Adjusted R-squared is high enough. With regard to the risk hedging strategy for an administrator, the ECM model can provide the most effective risk hedging.

Table 3: Comparisons between hedging models

	<b>OLS</b>	<b>ECM</b>	<b>GARCH (1,1)</b>	<b>EGARCH (1,1)</b>
Hedge ratios	0.821817*	0.953025*	0.930061*	0.928500*
Adjusted R <sup>2</sup>	0.703496	0.868306	0.691204	0.691550

\*Significant at 5% level.

## 4 Conclusions

This paper estimated optimal hedge ratios and examined the hedging effectiveness of the BTC Future using alternative models, both constant and time varying, over the period from January 2018 to March 2021. The findings of this study suggest that in terms of risk reduction the ECM is the appropriate method for estimating optimal hedge ratios as it provides better results than the conventional OLS method, the ECM with GARCH errors, the GARCH model and the EGARCH (1,1) model. The evidence presented in this paper strongly suggests that the BTC futures contract is an effective tool for hedging risk. Hence, the introduction of this contract has given portfolio managers and investors a valuable financial instrument by which they can avoid risk at times they wish to do this without liquidating their spot position or changing their portfolios composition. Moreover the BTC futures contract can be safely accepted by risk managers as an effective instrument for risk management of BTC positions.

Futures directions could include studies on the correlation of BTC futures with other existing futures in order to examine how the contract behaves in a portfolio context and how its inclusion in a multi asset portfolio can affect cross margining.

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