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Impact of Supply Risk Sharing on Supply Chains: Multiplicative vs. Additive Risks

Junhyun Bae[1*] and Ji-Hung (Ryan) Choi[2]

Abstract

We consider a supply chain in which a retailer orders from a manufacturer(s) who face(s) a stochastic supply risk (random yield) under single or dual-sourcing cases. In specific, we look into this problem in two different yield risks: multiplicative and additive. One might intuit that if the retailer shares a manufacturer's random yield risk with the manufacturer, the manufacturer will be better off. Interestingly, we confirm that this intuition is only valid in the additive yield risk but not necessarily in the multiplicative yield risk. Moreover, under dual sourcing, both manufacturers fiercely compete on their prices (i.e., Bertrand-like competition) to become the sole source in the additive yield risk, but the manufacturers do not compete as much in the multiplicative yield risk. Lastly, this paper shows that the supply chain inefficiency may decrease (increase) as risk-sharing increases in the additive risk model under dual sourcing (single sourcing) while it does not change in the multiplicative risk model.

Key words: supply risk, random yield, dual sourcing, game theory
JEL Classification: L11, M11, M21

[1](Corresponding author) School of Business Administration, Oakland University, Rochester MI 48309, USA. Email: bae@oakland.edu
[2]College of Business, Eastern Michigan University, Ypsilanti, MI 48197, USA. E-mail: jchoi20@emich.edu

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1 Introduction

Supply risk has been of great concern to firms in a supply chain, particularly during the Covid-19 period. Among various supply risk factors, random yield is critical and inevitable in various industries and processes, including the semiconductor industry, the agricultural industry, the chemical industry, and the transportation process (Keren 2009, Xie et al. 2021). Due to the nature of random yield, various problems may arise in reality. For example, farmers sow seeds, but they do not know how many plants they can harvest; semiconductor chips may be subject to even little contamination, which leads to yield loss; electronic devices or automobile vehicles should go through a quality test to meet a customer's demand, which is seldom 100% perfect, and some deliveries may not be made due to severe weather conditions or geographical risk in certain areas. Among such different types of random yields, one type would describe the actual yields depending on the order quantity, such as defective rates, but the other would describe the ones independent of the order quantity, such as supply shocks in certain areas.

In this study, we analyze and compare two different types of yield models, namely multiplicative and additive random yields, to understand the firm's decisions in a supply chain setting. On the one hand, the multiplicative random yield can occur mainly due to endogenous factors within the supply chain, such as inherent process limitations and imperfect labor skills. As mentioned previously, the yield outcome is directly proportional to the initial order quantity, and therefore, the *absolute* yield loss gets bigger as the order size increases. On the other hand, the additive random yield can occur due to exogenous factors outside the supply chain, such as disease and weather in the agricultural industry (Keren 2009), customs clearance delay (Dalzell et al. 2020), and labor shortages (Smith 2023). In this case, the *absolute* yield amount is independent of the order size.[1]

Numerous papers in the operations management (OM) literature have discussed random yield under various contexts, such as offshore/onshore sourcing (Jung 2020), dual sourcing (Wang et al. 2010, Tang and Kouvelis 2011), aggregate farming (An et al. 2015), dual channel (Niu et al. 2019), price postponement and risk aversion (Kouvelis et al. 2021), and supply chain coordination (Li et al. 2013). For example, Tang and Kouvelis (2011) study the benefits of supplier diversification for dual-sourcing duopolists with a proportional random yield. Following this model, An et al. (2015) discuss the effects of forming cooperatives in the agricultural sectors. In addition, Jung (2020) investigates the firm's sourcing strategies under supply and demand uncertainty and shows that risk pooling is an essential driver in supplier selection: offshoring, onshoring, or dual. Similarly, Niu et al. (2019) also study the dual-sourcing and dual-channel decisions with a production random yield in the presence of a competitive supplier. Typically, random yield papers assume that supply uncertainty is often represented as a multiplicative or proportional yield (see Yano and Lee (1995)). This means that the actual outcome of the yield (i.e., the delivered quantity) is proportional to the initial order quantity. Only a limited number of research papers address supply uncertainty using an additive yield approach (Keren 2009, Wang et al. 2014, Xiao and Wang 2023), where the actual yield outcome results in an absolute deduction from the initial order quantity. Most additive yield papers utilize the fixed-price model to discuss newsvendor and periodic review lot sizing problems. To the best of our knowledge, our work is the first attempt to understand the random yield in two ways: multiplicative and additive, with endogenous price-setting.

[1] While recognizing the possibility of encountering mixed random yield models in practical scenarios, we deliberately exclude such cases from our study. The reason behind this exclusion is that the combination of both multiplicative and additive yield models could obscure the fundamental objectives and focus of our research.

Using these yield models, we study how firms should make their decisions differently in a supply chain setting. Our research leads to the following important research questions: *Does any party in the supply chain have incentives to share random yield risk with other parties?* If so, *which party in the supply chain would receive more benefits?* and *does this result vary under different sourcing strategies: single and dual sourcing?*

To answer our research questions, this paper analyzes a supply chain with a retailer and a manufacturer(s). We investigate this setting in two-by-two cases: single and dual-sourcing, with two different yield models, multiplicative and additive. We find that the results are substantially different across the two models. One might intuit that if the retailer shares a manufacturer's random yield risk with the manufacturer, the manufacturer will be better off. Interestingly, we find that this intuition is only valid in the additive yield model but not in the widely-used multiplicative yield model. Moreover, under dual sourcing, both manufacturers fiercely compete on their prices (i.e., Bertrand-like competition) to become the sole source in the additive yield model, but the manufacturers do not compete as much in the multiplicative yield model. Lastly, this paper shows that the supply chain inefficiency may (not) decrease as risk-sharing increases in the additive model under dual sourcing (single sourcing) while it does not change in the multiplicative model. Our work sheds light on the firm's decisions when facing subtle different yield types in practice.

2 Model

We study a supply chain where a retailer orders from a manufacturer(s) who face(s) random yield. The following equations describe the profit functions of a retailer (denoted by R) and a manufacturer (denoted by M), respectively.

$$\pi_R = (a - f(q|\Omega))f(q|\Omega) - w((1-\gamma)f(q|\Omega) + \gamma q),$$
$$\pi_M = w((1-\gamma)f(q|\Omega) + \gamma q) - cq,$$

where $\Omega \in \{Add, Multi\}$ ("Add" as the additive risk scheme and "Multi" as the multiplicative risk scheme), w is a wholesale price, c is a production cost, q is an order quantity, and a is a potential market size. As in the literature, a product selling price is determined by $a - f(q|\Omega)$. Throughout the paper, we use $f(.|\Omega)$ as the amount of the actual shipments that were delivered or produced. We assume that all firms do not observe yield's realization, and the manufacturers have no incentive to deliver less than their outcomes $f(.|\Omega)$ (Tang and Kouvelis 2011). Consistent with the literature, we assume that the market size, a, should be sufficiently larger than c. In the multiplicative model, $f(q|Multi) = zq$ where a *random yield rate*, $z \in (0, 1]$, follows a mean of μ and a standard deviation of σ_m. In the additive model, $f(q|Add) = q - \hat{z}$ where a *random yield loss*, $\hat{z} \in (0, q)$, follows a mean of ϵ and a standard deviation of σ_a.[2] This means a random yield loss, $\hat{z}$, should always be positive and not larger than the order quantity, q.

We also introduce $\gamma \in [0, 1]$, which indicates a risk-sharing factor. γ describes how much the retailer would share the cost of the unfilled order with the manufacturer. In the literature, researchers have assumed that either the downstream firm can take full responsibility for this uncertainty (Tang and Kouvelis 2011) or can pay only for what is delivered (Yu et al. 2009). The

[2] Note that when the *random yield rate* z increases in the multiplicative model, it results in a higher actual shipment being generated. Conversely, in the additive model, when the *random yield loss* $\hat{z}$ increases, a lower actual shipment is expected.

former corresponds to the case with $\gamma = 1$, and the latter corresponds to the case with $\gamma = 0$. We parameterize this risk-sharing level to see how all the results vary with the level of risk-sharing. This risk-sharing concept can be interpreted in an alternative manner. For example, a firm can have options to obtain a full refund for what is not delivered as promised or defective products.

Under the dual-sourcing case, the profit functions of a retailer and a manufacturer $i \in \{1, 2\}$ should follow

$$\pi_R = (a - f(q_1, q_2|\Omega))f(q_1, q_2|\Omega) - w_1((1-\gamma)f(q_1|\Omega) + \gamma q_1) - w_2((1-\gamma)f(q_2|\Omega) + \gamma q_2),$$
$$\pi_{Mi} = w_i((1-\gamma)f(q_i|\Omega) + \gamma q_i) - cq_i,$$

where in the multiplicative model, $f(q_1, q_2|Add) = z_1 q_1 + z_2 q_2$ and $z_i \in (0, 1]$ with a mean of μ and a standard deviation of σ_m. In the additive model, $f(q_1, q_2|Add) = q_1 - \hat{z_1} + q_2 - \hat{z_2}$ and $\hat{z_i} \in (0, q_i)$ with a mean of ϵ and a standard deviation of σ_a. We make the following assumption to ensure that all the firms have incentives to play. $\mu \geq \frac{c}{a}$ and $\epsilon \leq \min\{\frac{(a-c)^2}{8c}, \frac{\sqrt{4(3a^2+c^2-3\sigma_a^2)}-(3a+c)}{6}\}$. This assumption illustrates that the yield rate (μ) should be large enough, and the production cost and yield variability should not be too large compared to the random yield loss magnitude (i.e., ϵ).

The game sequence is as follows: the manufacturer(s) decides its (their) wholesale price(s), and based on its wholesale price(s), the retailer makes an order quantity decision. Lastly, the manufacturer(s) will produce products upon the retailer's request under one of two different yield schemes. We assume that all the information is common knowledge.

3 Analysis

In this section, we analyze two-by-two cases: single and dual sourcing, with two different models, multiplicative and additive risks.

3.1 Single sourcing

We study the decentralized supply chain with single sourcing first.

***Multiplicative model*:** By backward induction, we start solving the retailer's problem. With the multiplicative model, the first-order condition becomes

$$a\mu - w(\gamma + (1-\gamma)\mu) - 2q(\mu^2 + \sigma_m^2) = 0.$$

It is easy to check the second-order condition is negative. By solving the equation above, we can get the following best response,

$$q(w) = \frac{a\mu - w(\gamma + (1-\gamma)\mu)}{2(\mu^2 + \sigma_m^2)}.$$

After plugging $q(w)$ into the manufacturer's profit function, we solve the manufacturer's problem for w. That is,

$$w^* = \frac{a\mu + c}{2(\gamma + (1-\gamma)\mu)}.$$

Thus, the order quantity should be

$$q^* = \frac{a\mu - c}{4(\mu^2 + \sigma_m^2)}.$$

Finally, we can provide the expected retailer and manufacturer's profits.

$$\mathbb{E}[\pi_M] = \frac{(a\mu - c)^2}{8(\mu^2 + \sigma_m^2)}, \qquad \mathbb{E}[\pi_R] = \frac{(a\mu - c)^2}{16(\mu^2 + \sigma_m^2)}.$$

***Additive model*:** By backward induction, we start solving the retailer's problem. The first-order condition yields

$$a - 2q - w + 2\epsilon = 0.$$

It is easy to check the second-order condition is negative. By solving the equation above, we can get the following,

$$q(w) = \frac{a - w + 2\epsilon}{2}. \tag{1}$$

After plugging $q(w)$ into the manufacturer's profit function, we solve the manufacturer's problem for w. Solving the equation above for w yields the following.

$$w^* = \frac{a + c + 2\gamma\epsilon}{2}. \tag{2}$$

Thus, the order quantity should be

$$q^* = \frac{a - c + 2(2-\gamma)\epsilon}{4}.$$

Finally, we provide the expected retailer and manufacturer's profits.

$$\mathbb{E}[\pi_M] = \frac{(a-c)^2}{8} + \frac{\gamma\epsilon(a + \gamma\epsilon)}{2} - \frac{c\epsilon(2-\gamma)}{2}, \qquad \mathbb{E}[\pi_R] = \frac{(a-c)^2}{16} - \frac{\gamma\epsilon(3a + c + 3\gamma\epsilon)}{4} - \sigma_a^2.$$

We compare two different models shown above to provide interesting insights in terms of risk-sharing factors, random yield, and variability. Static comparisons derive the following proposition:[3]

Proposition 1. *The risk-sharing factor does not affect the profits for both parties in the multiplicative model. In the additive model, however, the more the retailer shares the manufacturer's risk, the higher (lower) profit the manufacturer (retailer) would earn.*

In the multiplicative model, as γ does not affect both parties' profits and order quantity, double marginalization does not change in γ. The manufacturer has no incentive to increase its wholesale price when γ increases as it will make the retailer significantly decrease the order quantity. It is intuitive to think that increasing risk-sharing would help the manufacturer earn more profits, but interestingly, in the widely-used multiplicative model, we do not observe this phenomenon. Thus, the risk-sharing factor in this case does not affect the supply chain inefficiency.

On the other hand, as the order quantity decreases in γ in the additive model, it is intuitive to see that increasing risk-sharing does not improve the retailer's profit but instead makes only the manufacturer better off. The retailer's optimal order quantity is solely determined by the manufacturer's wholesale price, w, which leads the manufacturer to take advantage of this by increasing its wholesale price. As the order quantity decreases in γ, the total supply chain profit also

[3] All proofs are in the Appendix.

decreases in γ.[4] This suggests that the risk-sharing factor remains ineffective in alleviating supply chain inefficiency (i.e., increasing the effect of double marginalization), but only the manufacturer would receive more profits.

Now, we compare the multiplicative and additive models in terms of random yield. In practice, one could think that the more yield loss, the more profit loss. Our models also well explain this intuition. However, in the additive model, the manufacturer's profit is not always worse off with respect to the yield loss. Our result shows that if γ is sufficiently high, the manufacturer's profit will increase with even the yield loss. This is because the manufacturer wants to impose a higher wholesale price with higher γ to extract more profits from the retailer. For the manufacturer, therefore, charging a higher wholesale price is a means of compensating itself for a random yield loss.

Proposition 2. *The higher the random yield rate (μ) the manufacturer has, the higher profits both retailer and manufacturer can achieve in the multiplicative model. Likewise, in the additive model, the retailer can always achieve a higher profit with a lower random yield loss (ϵ), but the manufacturer can achieve a higher profit with a lower random yield loss (ϵ) only when the risk-sharing factor (γ) is sufficiently high.*

One of the distinct differences between multiplicative and additive models is that the yield loss depends on its order quantity in the multiplicative model but not in the additive model. It is intuitive that a higher yield rate benefits the manufacturer in the multiplicative model. In the additive model, however, a higher yield loss might not harm the manufacturer's profit. Generally, the manufacturer's profit declines when faced with a significant yield loss. However, as Proposition 1 suggests, the retailer's risk-sharing mitigates the manufacturer's profit loss resulting from a high yield loss, thereby compensating for it in the multiplicative model.

Lastly, in the multiplicative model, yield variability lowers both firms' profits. On the other hand, increasing yield variability lowers the retailer's profit, not the manufacturer's, in the additive model. Yield variability plays an important role in the order quantity decisions in the multiplicative model, which impacts both parties' profits. In the additive model, yield variability is independent of the order quantity, and it appears only in the retailer's profit function.

3.2 Dual sourcing

This section studies the dual-sourcing case. That is, one retailer receives homogeneous products from two identical manufacturers. By backward induction, we start solving the retailer's problem first.

***Multiplicative model*:** With the multiplicative model, the first-order condition yields

$$a\mu - w_i(\gamma + (1-\gamma)\mu) - 2q_i(\mu^2 + \sigma_m^2) - 2q_j(\mu^2 + cov(z_1, z_2)) = 0$$

It is easy to check that the profit function is jointly concave in q_1 and q_2. By solving the equation above, we can get the following,

$$q_1(w_1, w_2) = \frac{a\mu(cov(z_1,z_2) - \sigma_m^2) + (\gamma + (1-\gamma)\mu)(\mu^2 + \sigma_m^2)w_1 - (\gamma + (1-\gamma)\mu)(\mu^2 + cov(z_1,z_2))w_2}{2(cov(z_1,z_2) - \sigma_m^2)(cov(z_1,z_2) + 2\mu^2 + \sigma_m^2)},$$

$$q_2(w_1, w_2) = \frac{a\mu(cov(z_1,z_2) - \sigma_m^2) - (\gamma + (1-\gamma)\mu)(\mu^2 + \sigma_m^2)w_2 + (\gamma + (1-\gamma)\mu)(\mu^2 + cov(z_1,z_2))w_1}{2(cov(z_1,z_2) - \sigma_m^2)(cov(z_1,z_2) + 2\mu^2 + \sigma_m^2)}.$$

[4] $\frac{\partial \mathbb{E}[\pi_M + \pi_R]}{\partial \gamma} = -\frac{1}{4}\epsilon(a - c + 2\gamma\epsilon) < 0$, so it is obvious to see that the total expected profit decreases in γ.

Solving the manufacturer's problem, we find the equilibrium w_i as follows:

$$w_1^* = w_2^* = \frac{a\mu(\sigma_m^2 - cov(z_1, z_2)) + c(\mu^2 + \sigma_m^2)}{(\gamma + (1-\gamma)\mu)(\mu^2 + 2\sigma_m^2 - cov(z_1, z_2))}.$$

From the wholesale prices in equilibrium, we see that even when two manufacturers sell *homogeneous* products, they do not follow *Bertrand-like* price competition that both manufacturers are supposed to race toward the bottom by offering the lowest wholesale price possible. That is, the retailer will buy both products even if one's wholesale price is less than the other. Plugging these wholesale prices back into the $q_i(w_1, w_2)$ yields the order quantity as follows:

$$q_1^* = q_2^* = \frac{(a\mu - c)(\mu^2 + \sigma_m^2)}{2(\mu^2 + 2\sigma_m^2 - cov(z_1, z_2))(2\mu^2 + \sigma_m^2 + cov(z_1, z_2))}.$$

It is interesting to see that q_i^* decreases when $cov(z_1, z_2)$ goes to either σ_m^2 or $-\sigma_m^2$ if $\mu^2 = \sigma_m^2$. Note that $|\sigma_m^2| \geq cov(z_1, z_2)$. If $\mu^2 = \sigma_m^2$, the order quantity will be minimum when the two manufacturers' random yields are independent. If $\mu^2 \neq \sigma_m^2$, then q_i^* increases when $cov(z_1, z_2)$ goes to $\sigma_m^2 - \mu^2$. Finally, we can provide the expected retailer and manufacturer's profits. For all $i \in \{1, 2\}$,

$$\mathbb{E}[\pi_{Mi}] = \frac{(a\mu - c)^2(\sigma_m^2 - cov(z_1, z_2))(\mu^2 + \sigma_m^2)}{2(2\mu^2 + \sigma_m^2 + cov(z_1, z_2))(\mu^2 + 2\sigma_m^2 - cov(z_1, z_2))^2},$$
$$\mathbb{E}[\pi_R] = \frac{(a\mu - c)^2(\mu^2 + \sigma_m^2)^2}{2(2\mu^2 + \sigma_m^2 + cov(z_1, z_2))(\mu^2 + 2\sigma_m^2 - cov(z_1, z_2))^2}.$$

From all the equations above, we find that all the results from Proposition 1 and 2 are carried over under dual sourcing. That is, under dual sourcing, in the multiplicative model, the risk-sharing factor does not affect the profits for both parties. The higher random yields the manufacturer has, the higher profits both retailer and manufacturer can achieve[5]. Lastly, our result clearly shows that the total amount of units shipped with dual sourcing is larger than the one with single sourcing, which is robust to the literature.

***Additive model*:** By backward induction, we start solving the retailer's problem. The first-order condition yields

$$a - 2q_1 - 2q_2 - w_1 + 4\epsilon = 0,$$
$$a - 2q_1 - 2q_2 - w_2 + 4\epsilon = 0.$$

From the equations above, we can see that the retailer will choose any manufacturer offering the lowest wholesale price. If the retailer chooses only one firm with the lowest price, this game becomes a single-sourcing game as described in Section 3.1. Since there is only single sourcing, the best response of q for w should be Equation 1. By backward induction, the optimal wholesale price under single sourcing is the same as Equation 2, which we will denote by w^{1*} in this section. We denote this dual-sourcing case with one manufacturer to be $1F$. The retailer's profit function is as follows, after plugging in q^* derived from Equation 1.

$$\mathbb{E}[\pi_{R1F}^*] = \frac{1}{4}(a - w)^2 - w\gamma\epsilon - \sigma_a^2.$$

[5] In the multiplicative model, we find the following results: $\frac{\partial q_i}{\partial \gamma} = 0$, $\frac{\partial w_i}{\partial \gamma} < 0$, $\frac{\partial \pi_{Mi}}{\partial \gamma} = 0$, $\frac{\partial \pi_R}{\partial \gamma} = 0$, $\frac{\partial p}{\partial c} = \frac{4\mu(\mu^2+\sigma_m^2)}{(3\mu^2+4\sigma_m^2-cov(z_1,z_2))(3\mu^2+4\sigma_m^2+cov(z_1,z_2))}$, $\frac{\partial w}{\partial c} = \frac{(\mu^2+\sigma_m^2)}{(\gamma+(1-\gamma)\mu)(\mu^2+2\sigma_m^2-cov(z_1,z_2))}$.

If the two same wholesale prices are offered, then the retailer will split the order. In this case, to find closed-form solutions, we assume that the retailer utilizes the uniform allocation rule (i.e., $q_1 = q_2 = q$).[6] We denote this dual-sourcing case with two manufacturers to be $2F$. Let us examine the retailer's profit given any w under $2F$ (i.e., $w_1 = w_2 = w$), which follows the equation below.

$$\pi_{R2F} = (a - q_1 + \hat{z}_1 - q_2 + \hat{z}_2)(q_1 - \hat{z}_1 + q_2 - \hat{z}_2) \\ - w_1((1-\gamma)(q_1 - \hat{z}_1) + \gamma q_1) - w_2((1-\gamma)(q_2 - \hat{z}_2) + \gamma q_2).$$

Taking a derivative of the expected profit with respect to q yields the following:[7]

$$q_1^* = q_2^* = q^* = \frac{a - w + 4\epsilon}{4}.$$

By plugging these optimal q^* into the retailer's profit, we can find the following expected profit given $w_1 = w_2 = w$:

$$\mathbb{E}[\pi^*_{R2F}] = \frac{1}{4}(a-w)^2 - 2w\gamma\epsilon - 4\sigma_a^2.$$

Simply comparing $\mathbb{E}[\pi^*_{R1F}]$ and $\mathbb{E}[\pi^*_{R2F}]$, it is easy to see that given any w, $\mathbb{E}[\pi^*_{R1F}]$ gives us a higher profit. That is, regardless of the wholesale prices given by the manufacturers, the retailer will choose only single-sourcing. This leads to the following response from each retailer,

$$(q_1, q_2) = \begin{cases} (\frac{a-w_1+2\epsilon}{2}, 0), & \text{if } w_1 < w_2, \\ (q_1^*, q_2^*), & \text{if } w_1 = w_2, \\ (0, \frac{a-w_2+2\epsilon}{2}), & \text{if } w_1 > w_2, \end{cases} \tag{3}$$

where (q_1^*, q_2^*) such that $(\frac{a-w_1+2\epsilon}{2}, 0)$ or $(0, \frac{a-w_2+2\epsilon}{2})$ will arise with 50% of chance each.

This result implies that given the same wholesale prices offered by the manufacturers, the retailer's profit will be better off by single sourcing rather than dual sourcing as the yield loss will double in the additive model with two manufacturers. Moreover, in the additive model, the variability of the yield loss will be magnified under dual sourcing. Although the retailer would want to choose only one manufacturer, this upstream firm's competition leads to different results from the one in the single-sourcing case.

The additive yield case under dual sourcing is interesting because, unlike the results from the multiplicative model, the manufacturers now must engage in fierce competition in order to become single sourcing. In this case, there are two possible profit functions for each manufacturer: one with only one manufacturer winning (i.e., we denote this by $\mathbb{E}[\pi_{M1F}]$) and the other one with two manufacturers with the same wholesale prices ($w_1 = w_2$) given (i.e., we denote this by $\mathbb{E}[\pi_{M2F}]$). Only one manufacturer will be randomly picked by the retailer when $w_1 = w_2$, so simply $\mathbb{E}[\pi^*_{M2F}] = \frac{\mathbb{E}[\pi^*_{M1F}]}{2}$.

In order to fully analyze this manufacturer's game, we introduce the following notation. The wholesale price such that the manufacturer earns zero profit is denoted by

$$w^{0*} = \frac{1}{2}(a + c + 2\gamma\epsilon \pm \sqrt{(a + c + 2\gamma\epsilon)^2 - 4c(a + 2\epsilon)}).$$

Using all of the functions and notations defined above, we can derive the following lemma.

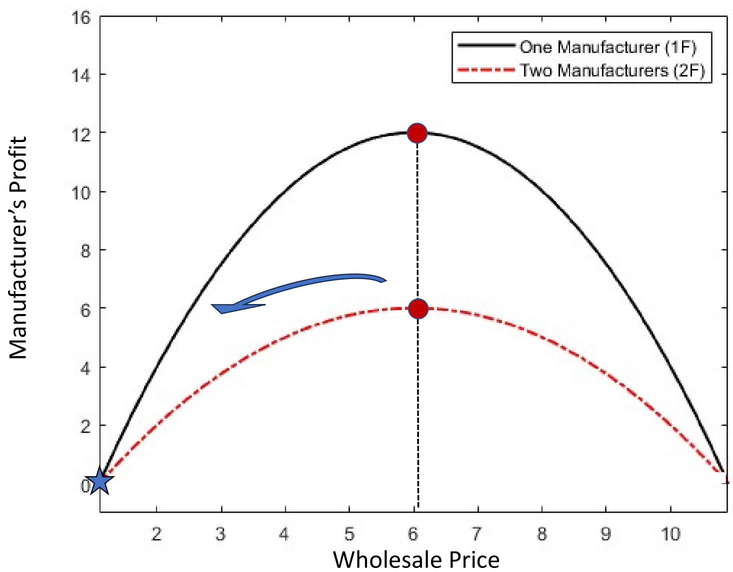
[6] We note that any different allocations will not change our main results.

[7] The second-order condition is negative.

Lemma 1. *In the dual-sourcing and additive model, both manufacturers optimally choose and offer their wholesale prices* $w^{0*} = \frac{1}{2}(a + c + 2\gamma\epsilon - \sqrt{(a + c + 2\gamma\epsilon)^2 - 4c(a + 2\epsilon)})$*. However, the retailer takes only one of the two manufacturers' offers and decides its order quantity* $q^* = \frac{1}{4}(a - c + 4\epsilon(1 - \gamma) + \sqrt{(a + c + 4\gamma\epsilon)^2 - 4c(a + 4\epsilon)})$.

In contrast to the single-source case, manufacturers reduce their wholesale prices in the dual-sourcing and additive model due to competition. This ultimately leads to minimum profits for the manufacturers while the retailer reaps all the benefits, which shows Bertrand-like competition. From Lemma 1, we can show that in equilibrium, the manufacturer's profit becomes zero and the retailer's profit follows:

$$\mathbb{E}[\pi_R^*] = \frac{1}{16}[(a - c - 2\gamma\epsilon + \sqrt{A})^2 - 8\gamma\epsilon(a + c + 2\gamma\epsilon - \sqrt{A}) - 16\sigma_a^2], \quad (4)$$
$$\text{where } A = (a + c + 2\gamma\epsilon)^2 - 4c(a + 2\epsilon).$$



Note that the parameters are $a = 10, c = 1$, $\gamma = 0.5$, and $\epsilon = 1$.

Figure 1: Illustration of Equilibrium in Additive Model

Figure 1 shows where the equilibrium points should be located from the manufacturer's perspective. The solid line indicates the manufacturer's *expected* profit when only one firm with a lower wholesale price was chosen, and the dotted line indicates the manufacturer's *expected* profit when two firms offer the same wholesale price and only one of them is chosen by the retailer. As two manufacturers fight over wholesale prices, one manufacturer is always incentivized to deviate by decreasing its wholesale price and moving up to the profit function under 1F. Then, just like Bertrand's game, two manufacturers will end up reaching the same wholesale price where zero profit is achieved (i.e., where the 'star' is illustrated in Figure 1) although the retailer would choose only one of them.

Using all the findings above, we find how the risk-sharing factor, γ, impacts the profits, q^*, and w^* under dual sourcing. The following proposition summarizes the results.

Proposition 3. *In the dual-sourcing and additive model, as the risk-sharing factor (γ) increases, the manufacturer's optimal wholesale price (w^*) decreases, and the retailer's optimal order quantity (q^*) increases. The retailer's profit increases in the risk-sharing factor (γ) while the manufacturer's profit remains zero.*

Based on our findings from the single-sourcing and dual-sourcing cases, we can conclude that the manufacturers' competition plays a pivotal role in the dual-sourcing and additive model. Specifically, the order quantity increases in the risk-sharing factor, γ, in the dual-sourcing case, as shown in Proposition 3, while it decreases in γ in the single-sourcing case. This result is mainly due to the fact that, in the dual-sourcing and additive model, manufacturers' competition is too intense. The wholesale prices will be determined solely by the manufacturers' competition, not the retailer's order quantity. Hence, the risk-sharing benefit for the manufacturers ironically intensifies the manufacturer's competition. Under this situation, the retailer exploits the manufacturers' competition by increasing their order quantities. We can clearly see that the upstream competition overturns the results we find from the single-sourcing case. Lastly, we can find that as the retailer's profit increases in γ and the manufacturer's profit remains the same, the total supply chain profit, in turn, increases in γ under dual sourcing. This leads to the conclusion that the risk-sharing factor can mitigate the supply chain inefficiency, and the retailer may have incentives to increase γ.

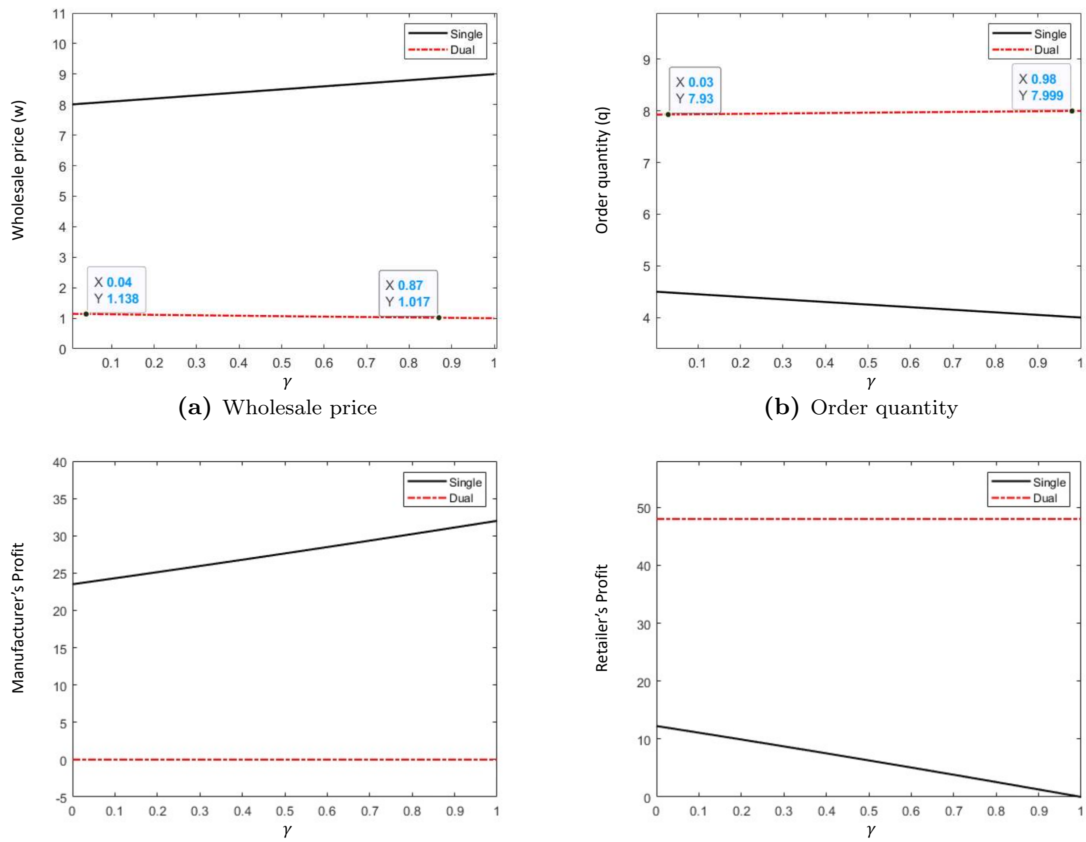
Figure 2 visually illustrates how the firm's performance would change in terms of γ, which analytically summarizes in Proposition 3. The solid (dotted) lines in Figures 2 indicate the firm's performance under single sourcing (dual sourcing). Interestingly, all these graphs show that the results are significantly different under two different sourcing strategies (i.e., single and dual sourcing). Recall that in the multiplicative model, all the main results remain consistent, regardless of single or dual sourcing. Therefore, we provide the following corollary, which can align with Proposition 1.

Corollary 1. *Under dual sourcing, the more the retailer shares the manufacturer's risk, the higher (constant zero) profit the retailer (manufacturer) would earn in the additive model. In the multiplicative model, however, the risk-sharing factor does not affect the profits for both parties.*

To recap, with single sourcing, the manufacturer's profit rises while the retailer's profit decreases due to the effect of γ. Now, in the context of dual sourcing and the additive case, the overall profit within the supply chain rises as the retailer's inclination to share risks increases while the manufacturer's profit remains zero. This indicates that in the case of dual sourcing with one manufacturer, the retailer's risk-sharing can effectively alleviate supply chain inefficiencies to a greater extent when compared to single sourcing.

4 Conclusion

This research examines a simple yet commonly observed situation in a supply chain: a random yield. We find some interesting policy and managerial implications as follows. First, with additive risk yield, the manufacturer may benefit from the retailer's risk-sharing under single sourcing, but



(a) Wholesale price **(b)** Order quantity **(c)** Manufacturers' profits **(d)** Retailer's profit

Note that the parameters are $a = 10, c = 1$, and $\epsilon = 1$.

Figure 2: Comparison of Firms' Performance in Additive Model

with multiplicative risk yield, the retailer's risk-sharing does not impact any firms in the supply chain. This result should be essential for policymakers because if any exogenous random risk occurs in the future, such as the Covid-19 pandemic or the Russia-Ukraine War, a risk-sharing mechanism could hurt the downstream firm's profit but not the upstream firm's. As the risk-sharing contract is not easily observed in public, it is fair to think that any government aid, including subsidy, should be first given to the one who is most vulnerable, in this case, the downstream firm.

In addition, we highlight that the supply chain inefficiency may be mitigated in both risk models under dual sourcing while the mechanism is significantly different. In the multiplicative model, the efficiency improvement can be possible due to the risk-pooling under dual sourcing, but in the additive model, the efficiency improvement can be possible due to the upstream firms' competition. This result concludes that managers would want to use dual-sourcing rather than single-sourcing in any risk models, which is consistent with the existing literature. Lastly, our work highlights that managers, in reality, may face a mix of these two random yields: additive and multiplicative, e.g., manufacturing semiconductors with delivery problems in a certain area. Thus, it will be crucial to understand how risk-sharing plays a role in contracting because the outcomes from our paper show a stark difference.

For future research, we recommend exploring scenarios involving a combination of multiplicative and additive yield models. It would also be valuable to investigate various situations, including downstream competition and order allocation games, by effectively incorporating both yield models. Although we do not cover how to implement the risk-sharing factor as a decision variable in our research, it would be interesting to see this topic under information asymmetry.

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Appendix

Proof of Proposition 1.
In the additive model, we find the following results: $\frac{\partial q^*}{\partial \gamma} < 0$, $\frac{\partial w^*}{\partial \gamma} = \epsilon > 0$, $\frac{\partial \mathbb{E}[\pi_M]}{\partial \gamma} > 0$, and $\frac{\partial \mathbb{E}[\pi_R]}{\partial \gamma} < 0$. In the multiplicative model, we find the following results: $\frac{\partial q^*}{\partial \gamma} = 0$, $\frac{\partial w^*}{\partial \gamma} < 0$, $\frac{\partial \mathbb{E}[\pi_M]}{\partial \gamma} = 0$, and $\frac{\partial \mathbb{E}[\pi_R]}{\partial \gamma} = 0$. $\square$

Proof of Proposition 2.
In the multiplicative model, we find the following results: $\frac{\partial \mathbb{E}[\pi_M]}{\partial \mu} > 0$, $\frac{\partial \mathbb{E}[\pi_R]}{\partial \mu} > 0$, $\frac{\partial^2 \mathbb{E}[\pi_M]}{\partial \gamma \partial \mu} = 0$, and $\frac{\partial^2 \mathbb{E}[\pi_R]}{\partial \gamma \partial \mu} = 0$. In the additive model, we find the following results: $\frac{\partial \mathbb{E}[\pi_M]}{\partial \epsilon} = \frac{1}{2}(-c(2-\gamma)+\gamma(a+2\gamma\epsilon))$, $\frac{\partial \mathbb{E}[\pi_R]}{\partial \epsilon} < 0$, $\frac{\partial \mathbb{E}[\pi_T]}{\partial \epsilon} < 0$, $\frac{\partial^2 \mathbb{E}[\pi_M]}{\partial \gamma \partial \epsilon} > 0$, $\frac{\partial^2 \mathbb{E}[\pi_R]}{\partial \gamma \partial \epsilon} < 0$, $\frac{\partial^2 \mathbb{E}[\pi_T]}{\partial \gamma \partial \epsilon} < 0$. $\square$

Proof of Lemma 1.
From Equation 3, we can find that the best response for each manufacturer is to choose w that is slightly smaller than the other manufacturer's wholesale price w. This game resembles the traditional Bertrand pricing game. The lower bound of w should be w^{0*} and the upper bound of w should be $\frac{1}{2}(a + c + 2\gamma\epsilon + \sqrt{(a + c + 2\gamma\epsilon)^2 - 4c(a + 2\epsilon)})$. $\square$

Proof of Proposition 3.
It is easy to check that $\frac{\partial w^{0*}}{\partial \gamma} = \frac{1}{2}(2\epsilon - \frac{2\epsilon(a+c+2\gamma\epsilon)}{\sqrt{(a+c+2\gamma\epsilon)^2-4c(a+2\epsilon)}}) \leq 0$ and, when $\gamma \in [0, 1]$, $\frac{\partial q^*}{\partial \gamma} = \frac{1}{4}(-4\epsilon + \frac{4\epsilon(a+c+4\gamma\epsilon)}{\sqrt{(a+c+4\gamma\epsilon)^2-4c(a+4\epsilon)}}) \geq 0$ is also true. The manufacturer's profit remains the same in γ. The retailer's profit ($\mathbb{E}[\pi_R^*]$) follows $\frac{1}{16}[-8\gamma\epsilon(a + c + 2\gamma\epsilon - \sqrt{A}) + (a - c - 2\gamma\epsilon + \sqrt{A})^2 - 16\sigma_a^2]$, where $A = (a + c + 2\gamma\epsilon)^2 - 4c(a + 2\epsilon)$. The derivative of the profit, $\mathbb{E}[\pi_R^*]$ w.r.t. γ derives $\frac{\partial \mathbb{E}[\pi_R^*]}{\partial \gamma} = \frac{\epsilon(a-c+2\gamma\epsilon-B)(a+c+2\gamma\epsilon-B)}{4B}$, where $B = \sqrt{-4c(a + 2\epsilon) + (a + c + 2\gamma\epsilon)^2}$. Let's suppose $\frac{\partial \mathbb{E}[\pi_R^*]}{\partial \gamma} > 0$. Then, $(a-c+2\gamma\epsilon-B)(a+c+2\gamma\epsilon-B) > 0$, which means that $(a-c+2\gamma\epsilon-B)$ and $(a+c+2\gamma\epsilon-B)$ should be either both positive or both negative. (i) First, assume that both $(a - c + 2\gamma\epsilon - B) > 0$ and $(a + c + 2\gamma\epsilon - B) > 0$. In this case, if the first inequality, $(a - c + 2\gamma\epsilon - B) > 0$, is true, the second inequality, $(a + c + 2\gamma\epsilon - B) > 0$, must hold as well. Therefore, we derive the following: $(a-c+2\gamma\epsilon-B) > 0 \Leftrightarrow a-c+2\gamma\epsilon > \sqrt{-4c(a + 2\epsilon) + (a + c + 2\gamma\epsilon)^2} \Leftrightarrow (a+2\gamma\epsilon-c)^2 > -4c(a+2\epsilon)+(a+c+2\gamma\epsilon)^2 \Leftrightarrow 4c(a+2\epsilon) > (a+c+2\gamma\epsilon)^2-(a+2\gamma\epsilon-c)^2 \Leftrightarrow 4ca+8c\epsilon > 4ac+8c\gamma\epsilon \Leftrightarrow 8c\epsilon > 8c\gamma\epsilon$. Since $0 \leq \gamma \leq 1$, the last inequality is true. Therefore, it proves that $\frac{\partial \mathbb{E}[\pi_R^*]}{\partial \gamma}$ is always positive. (ii) As proved in (i), there is no such a case where $(a - c + 2\gamma\epsilon - B) < 0$, because $(a - c + 2\gamma\epsilon - B)$ is always positive. Therefore, there is no need of further proof of the second case. Given the analysis in (i) and (ii) above, we conclude that the retailer's profit increases in its risk-sharing-factor, γ. $\square$

Proof of Corollary 1.
This proof is omitted.