Mega Publishing Limited
Bulletin of Applied Economics, 2017, 4(2), 1-36 | December 1, 2017

On the efficiency of various expansionary fiscal policies and cuts in taxation rates in order to sustain economic activity

Séverine MENGUY [1]

Abstract

We use a simple DSGE model in order to evaluate the efficiency of various fiscal policies intended to sustain economic activity and growth. A decrease in the consumption taxation rate appears as the most efficient fiscal policy. Indeed, as goods are then less expansive, it would imply an increase in the same proportion of all components of economic activity: private consumption and investment, as well as public expenditure. Besides, it would also strongly favor public investment in the composition of public expenditure, in order to increase the productivity of private factors and to satisfy the higher global demand. In comparison, a decrease in the capital taxation rate would reduce the capital cost, and it would favor private and public investment. However, the effect would be minor on private consumption and even negative on public consumption expenditure; the increase in global economic activity would then be more moderate. Finally, a decrease in the labor taxation rate would not be able to increase private economic activity, in the framework of our model, and it would favor public consumption to the detriment of the most productive public investment expenditure.

JEL classification numbers: E61, E62, H21
Keywords: DSGE model, budgetary policy, consumption taxation rate, capital taxation rate, labor taxation rate.

1 Introduction

Is public expenditure growth enhancing (positive fiscal multiplier in Keynesian models), or can fiscal consolidation be expansionary? The question has been largely studied in the economic literature, without having a clear-cut answer. First, De Haan and Romp (2005) mention the problem of reverse causality between capital public spending and economic growth. Indeed, public capital can improve the productivity of private factors, but in parallel, a higher productivity enhances economic growth and so, the demand and supply of public services. In the same way, Bouakez and Rebei (2007) develop a RBC model where households' preferences depend on public spending. Then, they show the Edgeworth complementary between private and public spending: public expenditure increases the marginal utility of private consumption, and so, a public spending shock also increases this private consumption (crowding-in effect). Nevertheless, economic studies give various results regarding the efficiency of public spending in order to sustain economic growth.

Indeed, these results strongly depend on the initial fiscal situation of the country: fiscal consolidations can increase private investment and economic growth, especially if the country is initially excessively indebted. They also depend on the structure of the fiscal consolidation: spending cuts (government wage bills, welfare payments, or unproductive expenditure) usually seem to be more

[1] Université Paris Descartes, Sorbonne Paris Cité, 12 rue de l'école de médecine, 75270 PARIS CEDEX 06, France.

Article Info: *Received*: May 23, 2017. *Revised*: July 30, 2017
Published online: December 1, 2017

growth-enhancing [Alesina and Ardagna (2010)]. So, after the 1970's, the efficiency of public expenditure in order to sustain economic growth by the way of budgetary multipliers higher than one has strongly been challenged. The latter could be smaller than one, and even negative, in a framework where the public debt is excessive and where the sustainability of public finances is put into question. Besides, consolidations based on spending cuts could imply non-Keynesian effects: the adjustment on the labor market and the decrease in production costs could increase profits and private investment.

More precisely, in New-Keynesian models, an increase in public expenditure increases global demand, labor demand, real wages (even if labor productivity is weaker), economic activity and private consumption. For example, using a DSGE model, Forni *et al.* (2009) underline the prevalence of empirical mild Keynesian effects of public expenditure. In particular, government purchases of goods and services, compensations for public employees or transfers to households would have small and short-lived expansionary effects on private consumption. The effects would be more significant on the revenue side: decreases in labor income and consumption tax rates would have sizeable effects on consumption and output, while a reduction in capital income taxation rate would favor investment and output in the medium run. Furthermore, Fatas and Mihov (2001) find that increases in public expenditure (particularly regarding wages of public employees) are followed by strong and persistent empirical increases in consumption and employment (the budgetary multiplier is larger than one).

In the same way, Pappa (2004) shows that shocks to government consumption and investment increase real wages and employment contemporaneously both in US aggregate and in US state data, in conformity with New-Keynesian predictions. Indeed, a government consumption shock, financed by a higher budgetary deficit, increases global demand (absorption effect), and thus also labor demand, real wages, employment and output. Moreover, this positive effect on economic activity is the highest in case of an investment shock, which appears as the most beneficial. Pappa (2009) also identifies fiscal shocks in the United-States, between 1969 and 2001, with a VAR methodology, using the hypothesis that fiscal shocks raise output and the budgetary deficit. Then, she shows that an increase in the public deficit as well as in public consumption or investment empirically increases real wages and employment (the evidence for public employment shocks is mixed).

On the contrary, Real Business Cycles models anticipate that increases in government spending should imply a decrease in labor productivity and in real wages; indeed, more resources are then absorbed by the government. Besides, the standard negative wealth effect implies that households feel poorer because of the decrease in their permanent income. Therefore, economic agents should increase their work effort and their labor supply, whereas they should reduce their consumption. For example, Burnside *et al.* (2004) show that fiscal shocks (increase in military purchases) increase capital and labor income tax rates, they increase aggregate hours worked, but they decrease real wages. They are also associated with short lived rises in aggregate investment and small declines in private consumption. In the same way, Edelberg *et al.* (1999) show that an exogenous increase in US government purchases increases the present value of the tax burden for the representative household, implies a negative wealth effect, and increases labor supply. So, employment, output, the real interest rate and nonresidential investment (capital which is substitutable to labor in the production process) rise, while real wages, residential investment (consumption in housing services) and private consumption expenditures fall.

Therefore, eventually, the question of the size and even of the sign of the budgetary multiplier is not clear cut, neither theoretically nor empirically.

To contribute to this debate, the current paper uses a DSGE model with a detailed fiscal block, in order to study the effect of the structure of public expenditures and resources. Indeed, public spending has various degrees of productivity, as some public expenditure can strongly enhance economic growth, whereas some public expenditure is quite inefficient and unproductive. Different types of public expenditure have various impacts on private factors (capital and labor) marginal productivity. Some are highly productive, in particular investment in capital: highways, airports, electric and gas facilities, water systems, etc. Some are moderately productive (education, healthcare), whereas others are quite weakly productive (entertainment, culture, national defense and environment, social transfers). Besides, regarding fiscal resources, we must also distinguish between various types of distortionary taxes: on consumption, on capital or on labor. Thanks to this detailed fiscal framework, we will then be able to shed light on the efficiency of various fiscal instruments and of variations in different taxes in order to really sustain economic growth.

The rest of the paper is organized as follows. The second section reminds the numerous factors affecting the efficiency of the fiscal policy in order to sustain economic growth mentioned in the economic literature. The third section describes our DSGE model: economic agents, monetary and budgetary policies. The fourth section studies the consequences of variations in various taxation rates when the budgetary policy is passive and conducts the optimal budgetary policy without taking into account the necessity to balance the budget. The fifth section introduces the implications of the necessity, for the government, to balance the budget and to avoid an out-bidding of the indebtedness level. Finally, the sixth section concludes the paper.

2 Economic literature

The first parameter affecting the efficiency of an expansionary budgetary policy is, obviously, the nature of the global policy-mix: indeed, the budgetary policy is all the more efficient as monetary policy is more constrained and less active. For example, Sims and Wolff (2013) study the state-dependence of the output and welfare effects of shocks to government purchases in a DSGE model with real and nominal frictions and a rich fiscal financing structure. Then, in an historical simulation, the authors show that the output multiplier is quite high: from 1 to 1.5, it would reach about 2 in a Zero Lower Bound (ZLB) framework (inactive monetary policy). Nevertheless, an excessively high level of government spending would tend to decrease aggregate welfare, and it could therefore often be counter-productive.

In the same way, in the framework of a medium scale DSGE model, Zubairy (2014) estimates the effects of a discretionary fiscal policy. The multiplier for government spending would be 1.07, as higher public spending is able to boost private consumption in the short run, but its effect would be quickly decreasing in the long run. The size of this multiplier is also increased if monetary policy is more accommodative and if it is less contractionary after an increase in public expenditure. Furthermore, a cut in capital tax or labor tax of 1% would imply an increase in GDP of 0.34% and 0.13% respectively, but the stimulating effect on investment would take more time to be observed. Davig and Leeper (2011) also underline that an increase in government purchases is all the more detrimental to economic growth as monetary policy is active. Indeed, the nominal interest rate then increases more than proportionally to inflationary expectations, and the increase in the real interest rate crowds-out private consumption. On the contrary, if monetary policy is more passive, the decrease in the real interest rate implies a much larger multiplier on economic activity. In the same way, Leeper *et al.* (2011) show that fiscal multipliers are larger if monetary policy is more accommodative, and in a ZLB framework where monetary policy is constrained to be more passive. They are also larger if the economy is more closed, as multipliers tend to decrease in an open-economy framework, and if the proportion of non-savers (non-Ricardian and constrained consumers) is higher.

So, another parameter affecting the budgetary policy efficiency is the nature of the representative household, and the possibility to take into account non-Ricardian and constrained consumers. For example, Galí *et al.* (2007) introduce the existence of rule-of-thumb consumers in a New-Keynesian model, and they also consider sticky prices to study the effect of government spending on consumption. So, in the framework of an inter-temporal budgetary constraint of infinity-lived Ricardian households, if the budgetary deficit must later be financed, public spending crowds-out private consumption. On the contrary, in the framework of non-Ricardian households constrained to consume their current income, the budgetary multiplier can be higher than one. Besides, the authors mention that empirical evidence according to quarterly US data over the period 1954-2003 would mainly support this hypothesis (multiplier around 1.74 at the end of the second year). Therefore, the authors underline the necessity to take into account this existence of non-Ricardian consumers, in order to reconcile the theoretical results with the empirically observed expansionary consequences of higher public expenditure.

Drautzberg and Uhlig (2011) or Forni *et al.* (2009) also find that the fraction of transfers given to rule-of-thumb consumers improves the efficiency of an increase in public expenditure to sustain economic activity. However, Coenen and Straub (2005) revisit the effects of government spending shocks on private consumption within an estimated New-Keynesian DSGE model of the euro area. They show that the presence of non-Ricardian households (consuming current income as consumption smoothing is impossible because they are liquidity constrained) is in general conducive to raising the level of consumption in response to government spending shocks. Nevertheless, the latter would usually not crowd-in private consumption, because the estimated share of non-Ricardian households is relatively low, and because the highly persistent nature of government spending shocks induces large negative wealth effects, inducing households to work more but to consume less. Therefore, the authors are really doubtful about the efficiency of an expansionary budgetary policy in order to sustain economic growth.

Another important parameter is the nature of the expansionary budgetary policy conducted by the government. Indeed, cuts in taxation rates are often found to be more expansionary than increases in public expenditure. For example, Alesina and Ardagna (2010) examine the evidence on episodes of large stances in fiscal policy (politically motivated modification in the budgetary deficit), both in cases of fiscal stimuli (to increase GDP) and in cases of fiscal adjustments (to reduce the public debt-to-GDP ratio) in OECD countries from 1970 to 2007. So, they find that fiscal stimuli based upon tax cuts are more likely to increase growth than those based upon spending increases. As for fiscal adjustments, those based upon spending cuts and no tax increases are more likely to reduce deficits and debt over GDP ratios than those based upon tax increases. In the same way, Ardagna (2004) uses data from a panel of OECD countries, and he shows that the success of fiscal adjustments in decreasing the debt-to-GDP ratio depends on the size of the fiscal contraction. Besides, whether a fiscal adjustment is expansionary depends largely on its composition. In particular, stabilizations implemented by cutting public spending would lead to higher GDP growth rates. Furthermore, the effects of the composition of the fiscal adjustment on economic growth would work mostly through the labor market rather than through agents' expectations of future fiscal policy.

Coenen *et al.* (2008) use the ECB’s New Area-Wide Model (NAWM) to model fiscal consolidation as a permanent reduction in the targeted government debt-to-output ratio. Then, they find that fiscal consolidation has positive long-run effects on key macroeconomic aggregates such as output and consumption, notably when the resulting improvement in the budgetary position is used to lower distortionary taxes. At the same time, fiscal consolidation gives rise to noticeable short-run adjustment costs, whereas some fiscal instruments may have pronounced distributional effects, to the extent that households differ with regard to their ability to participate in asset markets and to their dependence on fiscal transfers. Mertens and Ravn (2011) also show that, in the framework of a DSGE model, ‘anticipated’ tax cuts may be contractionary. However, after their implementation, ‘surprise’ and effective exogenous tax cuts have expansionary and persistent effects on output, consumption, investment and hours worked. Indeed, if labor taxation rates are reduced, real net wages increase, as well as labor supply due to the intra-temporal substitution effect. There is then a temporary and gradual rise in hours worked as well as in private consumption. A decrease in capital taxation rates also discourages saving and increases current consumption and production.

In the same way, according to Ludvigson (1996), deficit financed cuts in distortionary income taxation may stimulate investment and be expansionary, even if agents expect future taxes on capital income to be higher. Indeed, the fiscal shock implies a substitution from leisure to labor (the elasticity of labor supply is a fundamental parameter), increasing output. Besides, higher future capital taxes decrease the returns on sparing and increase consumption levels. Therefore, an increase in government expenditure financed with distortionary taxation may decrease investment, output and consumption, whereas it can be expansionary if it is financed with a budgetary deficit. At least as long as this budgetary deficit is not too persistent on the public debt level, which could be detrimental to economic growth. Furthermore, Blanchard and Perotti (2002) characterize the dynamic effects of shocks in government spending and taxes on economic activity in the United States between 1947 and 1997, using a mixed structural VAR/event study approach. They show that positive government spending shocks have a positive effect on output, and positive tax shocks have a negative effect, even if the multipliers are moderate, often close to one. Nevertheless, both increases in taxes and increases in government spending have a strong negative effect on investment spending, which underlines the benefits of a tax consolidation policy.

Finally, in the framework of a DSGE model, Bhattarai and Trzeciakiewicz (2017) show that in the United-Kingdom, between 1987 and 2010, the most stimulating fiscal policy instrument is the consumption tax cut in the short term, the capital tax cut in the medium term, and the government investment expenditure in the long term. Nevertheless, the implications of fiscal policy depend significantly on the size of the nominal and real frictions. In particular, higher levels of price rigidity increase the government expenditure multipliers and exacerbate the tax multipliers. In addition, higher nominal wage rigidity tends to decrease tax multipliers in the long term.

Finally, a key parameter is, obviously, the productivity of the public spending realized by the government. Indeed, Carvalho and Martins (2011) show, that the cut in weakly productive public expenditure in order to increase highly productive public spending has expansionary consequences on investment and output. Therefore, fiscal consolidations are all the more beneficial as public expenditure that is cut is less productive (public current consumption, social transfers, public employment), and as the labor market is more competitive. Within a neo-classical model, Baxter and King (1993) also find that permanent changes in government purchases can lead to output multipliers exceeding one, and to larger effects than temporary changes. Besides, when it increases private factors productivity, public capital and investment can considerably increase private output and investment.

Furthermore, with a neo-classical growth model for the United-States between 1960 and 2008, Leeper *et al.* (2010) show that implementation delays for building public capital can produce small or even negative labor and output responses to increases in government investment in the short run. Afterwards, anticipated fiscal adjustments to deficit-financed spending matter both quantitatively and qualitatively for long-run growth effects. So, when public capital is insufficiently productive, distorting financing can make government investment contractionary at longer horizon. In the same way, Drautzberg and Uhlig (2011) insist on the delays for the effects of a fiscal stimulus to be observed, and on the negative effects of a distortionary taxation. Therefore, they find positive short run budgetary multipliers (around 0.51) but negative long run multipliers (around -0.42). So, the long run costs of a fiscal stimulus could be non-negligible. Besides, they show that the government investment component contributes to a positive multiplier, whereas the government consumption and transfer components lower the overall multiplier below zero.

Besides, Straub and Tchakarov (2007) underline that in the European Union, public consumption has increased since 1970 whereas on the contrary, public investment has been reduced. This could be detrimental, as both temporary and permanent public investment shocks generate larger fiscal multipliers than exogenous increases in public consumption. Thus, according to the authors, it should be beneficial to reverse this trend in the composition of public spending, in favor of safeguarding a higher level of public investment. Finn (1998) also distinguishes between two components of government spending: purchase of final goods and services (with expansionary consequences on private employment, investment and production) and public employment (with contractionary consequences). In this framework, he shows that distinguishing between public consumption and employment expenditure gives a smaller weight to public spending in order to influence cyclical fluctuations. However, the author also mentions that the shares of the various parameters regarding the public sector were sufficiently small, in the US economy between 1950 and 1993, to prevent a variation in these parameters to influence significantly the economic cycle.

More precisely, cuts in non-productive public spending can imply a fall in labor demand, and it can moderate wage claims. Therefore, this can decrease marginal production costs, increase profits and stimulate investment. Less consumption public expenditure implies a positive wealth effect, as less resources are absorbed by the government, and as public consumption is not perfectly substitutable with private consumption regarding household’s utility. There is then a decrease in private labor supply, whose average productivity improves, whereas private consumption increases. On the contrary, increases in productive public spending and public investment expenditure can improve the productivity of private factors, the real rental rate of capital, and it can imply a positive wealth effect crowding-in private investment. Public investment is useful and it must be preserved. Indeed, it increases not only aggregate demand, but also aggregate supply by enhancing global production and the marginal productivity of labor and private capital. Labor supply then decreases, whereas there is a positive effect on private consumption. Even if private production and investment are then initially ‘crowded-out’, as the productivity of capital progressively increases, more labor is also demanded; therefore, private investment, consumption and production finally increase.

Nevertheless, we must here mention the importance of the initial capital stock in the economy. Indeed, investment expenditure has a much higher productivity in under-developed countries, whereas its productivity is obviously much lower in the most developed countries, where the capital stock has already reached very high levels. This could explain why Perotti (2004), for example, finds a low efficiency of public investment expenditure. Indeed, using a structural Vector Auto-Regression approach, the author analyses the efficiency of various budgetary instruments to increase GDP, in Germany, the United-States, Australia, Canada and the United-Kingdom, between 1960 and 2001. Then, he shows that in the long run, the superiority and benefits of public investment expenditure, increasing the productivity of private production factors, is obvious, as spending multipliers are empirically always larger. However, Perotti (2004) finds no evidence that government investment shocks are more effective than government consumption shocks in boosting GDP. Indeed, even if the positive effects of government consumption are rather limited, public investment also appears to crowd-out private investment, especially in dwelling and in machinery and equipment, and it doesn’t always generate sufficient resources to finance its cost even in the long run.

3 The Model

In the framework of this abundant economic literature, the current paper aims at shedding light on the consequences of various fiscal policies in order to sustain economic growth. We consider a standard DSGE model, with a representative household and a representative firm. Regarding the budgetary authority, the contribution of the current paper is to take into account a developed fiscal block, where public consumption expenditure is distinguished from public investment expenditure, in order to study the consequences of the productivity of public spending on private consumption, production and economic growth. Besides, we also distinguish between the use of various fiscal resources and distortionary taxes: on consumption, capital or labor. In the rest of the paper, all lower-case letters denote variables in logarithms and in variations from their long run equilibrium values.

3.1 Households

Aggregate demand results from the log-linearization of the Euler equation, which describes the representative household’s expenditure decisions. In a given period (T), the representative household/consumer maximizes an inter-temporal utility function:

$$max \sum_{t=T}^{\infty} \beta^{t-T} E_T[U_t] \qquad (1)$$

Where: $E_t()$ is the rational expectation operator conditional on information available at date (t), and (β) is the time discount factor. Prices of goods, interest rates, taxation rates and wages are taken as given by the representative household.

We suppose that the utility function of a representative household is as follows:

$$U_t = \alpha_c \frac{\theta}{(\theta-1)} (C_t)^{\frac{(\theta-1)}{\theta}} + \alpha_g \frac{\theta}{(\theta-1)} (G_t)^{\frac{(\theta-1)}{\theta}} - \alpha_l \frac{1}{(1+\varphi)} L_t^{(1+\varphi)} \qquad (2)$$

The indices $(0<\alpha_c<1)$, $(0<\alpha_g<1)$ and $(0<\alpha_l<1)$ are the respective weights given to consumption of private goods, consumption of public goods and leisure in the utility function.

So, utility is an increasing and concave function of (C_t), an index of the household’s consumption of all goods that are supplied; (θ) is the elasticity of intertemporal substitution. Utility is also an increasing and concave function of real public goods and services provided in the home country (G_t). However, in this additive utility function, public consumption is supposed to be separable and it doesn’t affect the marginal utility of private consumption. Finally, utility is also a decreasing and convex function of the hours worked (L_t), where ($\varphi \geq 0$) is the inverse of the Frisch elasticity of labor supply, the inverse of the elasticity of the work effort with respect to the real wage.

This maximization is subject to a life time and inter-temporal nominal budget constraint, for whatever date (T) considered at which the actualization is realized. Regarding its expenditure, the representative household consumes goods (including taxes), it realizes investments in physical capital and it purchases government bonds. Capital is rented by households to firms, for which they receive a rental rate as well as profits which are all redistributed. Regarding its resources, the representative household receives labor and capital revenues (physical capital and profits), as well as gains from government bonds holding from the previous period. For simplicity, we suppose that these financial assets are only riskless one-period nominal government bonds. Besides, capital is not fully taxed, as we suppose that physical capital depreciation is exempted from taxation. We also avoid here lump-sum taxation and transfers made to households, as we suppose that both can offset each other. So, if we suppose complete financial markets, a household flow budget constraint for each period (T) takes the form:

$$(1+\tau_{c,T})P_T C_T + P_T INV_T + B_T = (1-\tau_{l,T})W_T L_T + (1-\tau_{k,T})R_T^k K_T + \delta\tau_{k,T} P_T K_T + P_T \Xi_T + (1+i_{T-1})B_{T-1} \quad (3)$$

With, in period (t): (C_t): real consumption; (INV_t): real investment in new physical capital; (K_t): stock of physical capital; (P_t): level of consumer prices; (W_t): nominal hourly wage; ($\tau_{l,t}$): taxation rate on labor income; ($\tau_{c,t}$): taxation rate on consumption; ($\tau_{k,t}$): taxation rate on capital; (R_t^k): nominal rental rate for capital services rent out to firms; (δ): depreciation rate of physical capital; (L_t): hours worked by the household; (i_t): nominal interest rate; (B_t): nominal value of riskless one period bonds (portfolio of all claims on the government) at the end of period (t); (Ξ_t) Nominal profits distributed to households by firms (dividends).

Summing equation (3) in order to obtain an intertemporal budgetary constraint, with:$\lim_{T\to\infty} B_T$=0, we have:

$$(1+\tau_{c,T})P_T C_T + P_T INV_T + E_T[\sum_{t=T}^{\infty}\frac{(1+\tau_{c,t+1})P_{t+1}C_{t+1} + P_{t+1}INV_{t+1}}{(1+i_t)\ldots(1+i_T)}]$$
$$= (1-\tau_{l,T})W_T L_T + (1-\tau_{k,T})R_T^k K_T + \delta\tau_{k,T}P_T K_T + P_T\Xi_t + (1-i_{T-1})B_{T-1}$$
$$+E_T[\sum_{t=T}^{\infty}\frac{(1-\tau_{l,t+1})W_{t+1}L_{t+1} + (1-\tau_{k,t+1})R_{t+1}^k K_{t+1} + \delta\tau_{k,t+1}P_{t+1}K_{t+1} + P_{t+1}\Xi_{t+1}}{(1+i_t)\ldots(1+i_T)}] < \infty \quad (4)$$

Furthermore, the capital stock varies according to the following equation:

$$K_{t+1} = (1-\delta)K_t + INV_t \qquad (5)$$

So, in logarithms and in terms of variations, the capital stock adjusts according to the following equation:

$$k_{T+1} = (1-\delta)k_t + \left(\frac{INV}{K}\right)inv_t = (1-\delta)k_T + \delta inv_T \quad (6)$$

where (δ) is the depreciation rate of capital.

Besides, the value of the real interest rate on capital is:

$$\left(\frac{R_t^k}{P_t}\right) = (\frac{1-\beta}{\beta})\frac{1}{(1-\tau_{k,t})} + \delta \qquad (7)$$

which implies: $\left(r_t^k - p_t\right) = -log(1-\widehat{\tau_{k,t}}) \sim \widehat{\tau_{k,t}}$

in logarithms and in variations, where a circumflex denotes a variation in the taxation rate.

Therefore, using equations (4), (5) and (7), the intertemporal budgetary constraint in a given period (T) is the following:

$$(1+\tau_{c,T})P_T C_T + E_T[\sum_{t=T}^{\infty}\frac{(1+\tau_{c,t+1})P_{t+1}C_{t+1}}{(1+i_t)\ldots(1+i_T)}]$$

$$= (1-\tau_{l,T})W_T L_T + P_T \Xi_t + E_T[\sum_{t=T}^{\infty} \frac{(1-\tau_{l,t+1})W_{t+1}L_{t+1} + P_{t+1}\Xi_{t+1}}{(1+i_t)\dots(1+i_T)}]$$
$$+(1-i_{T-1})B_{T-1} + \frac{1}{\beta}P_T K_T + E_T\{\sum_{t=T}^{\infty} \frac{[P_{t+1} - \beta(1+i_t)P_t]}{\beta(1+i_t)\dots(1+i_T)} K_{t+1}\} < \infty \qquad (8)$$

Current consumption and anticipated consumption for all future periods mustn't exceed current labor and capital (physical capital and profits) revenues and anticipated revenues for all future periods. Therefore, in this model, we allow for the possibility to borrow from one period to another, but we limit anticipated future revenues in order to avoid the possibility of Ponzi schemes[2].

In this context, the result of the maximization of equation (1) under the constraint (8) implies the following first order Euler conditions, regarding timing of expenditure decisions and inter-temporal substitution, for whatever period (T):

$$\frac{1}{(1+\tau_{c,T})P_T}\frac{\partial U_T}{\partial C_T} = \frac{\beta(1+i_T)}{(1+\tau_{c,T+1})P_{T+1}}\frac{\partial E_T(U_{T+1})}{\partial C_{T+1}} = \frac{\beta^k(1+i_{T+k-1})\dots(1+i_T)}{(1+\tau_{c,T+k})P_{T+k}}\frac{\partial E_T(U_{T+k})}{\partial C_{T+k}} \qquad (9)$$

Furthermore, by combining equations (2) and (9), (∀T), we have:

$$C_T = [\frac{(1+\tau_{c,T+1})E_T(P_{T+1})}{\beta(1+\tau_{c,T})(1+i_T)P_T}]^{\theta} E_T(C_{T+1}) \qquad (10)$$

So, in logarithms and in variations from their long run equilibrium values, with: log(1+x)~x provided (x) is sufficiently small; with $[\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \sim p_t - p_{t-1}]$: inflation rate; we have[3]:

$$c_T = E_T(c_{T+1}) - \theta[i_T - E_T(\pi_{T+1})] - \theta|\widehat{\tau_{c,T}} - E_T(\widehat{\tau_{c,T+1}})] \qquad (11)$$

Besides, for the representative agent in the country (i), we obtain the following optimal substitution between private consumption, public consumption and working time[4]:

$$\frac{\partial U_T}{\partial C_T} = -\frac{(1+\tau_{c,T})P_T}{W_T(1-\tau_{l,T})}\frac{\partial U_T}{\partial L_T} = \tau_{c,T}\frac{\partial U_T}{\partial G_T} \qquad (12)$$

Therefore, a higher real wage net of taxes reduces the marginal utility of leisure and increases the one of labor.

Besides, regarding labor supply, according to equations (2) and (12), in logarithms and in variations from their long run equilibrium values, we obtain[5]:

$$l_T = \frac{1}{\varphi}(w_T - p_T) - \frac{1}{\varphi}(\widehat{\tau_{l,T}} + \widehat{\tau_{c,T}}) - \frac{1}{\varphi\theta}c_T \qquad (13)$$

So, labor supply increases with the real wage, but it decreases with taxation rates ($\tau_{l,t}$ and $\tau_{c,t}$) and with the disutility of working time (φ).

---

2 Regarding returns on private capital, equation (8) implies that for n>T: $E_T(P_n K_n) = \frac{(1+\pi_n)(1+i_{n-1})\dots(1+i_T)}{[(1+\pi_n)-\beta(1+i_{n-1})]}P_T K_T$.

3 In the same way, by combining equations (1), (2) and (8), or by combining equations (11), (13) and (14), we can obtain, (∀T): $g_T = E_T(g_{T+1}) - \theta[i_T - E_T(\pi_{T+1})] - \theta|\widehat{\tau_{c,T}} - E_T(\widehat{\tau_{c,T+1}})]$
$l_T = E_T(l_{T+1}) + \frac{1}{\varphi}[i_T - E_T(\pi_{T+1})] - \frac{1}{\varphi}[\widehat{\tau_{l,T}} - E_t(\widehat{\tau_{l,T+1}})] + \frac{1}{\varphi}(w_T - p_T) - \frac{1}{\varphi}E_t(w_{T+1} - p_{T+1})$.

4 Here, we suppose that according to the budgetary constraint for the government, fiscal resources and expenditures vary in phase: for n≥0,
$\partial[\tau_{c,T+n}P_{T+n}C_{T+n} + \tau_{l,T+n}W_{T+n}L_{T+n} + \tau_{k,T+n}(R^k_{T+n} - \delta P_{T+n})K_{T+n}] = \partial(P_{T+n}G_{T+n})$.

5 $\frac{\partial U_T}{\partial L_T} = -\alpha_l L_T^{\varphi} = -\frac{W_T(1-\tau_{l,T})}{(1+\tau_{c,T})P_T}\frac{\partial U_T}{\partial C_T} = -\frac{W_T(1-\tau_{l,T})}{(1+\tau_{c,T})P_T}\alpha_c(C_T)^{-\frac{1}{\theta}}$.

Besides, regarding the variation in private consumption, according to equations (2) and (12), in logarithms and in variations from their long run equilibrium values, we obtain[6]:

$$g_T = c_T + \theta \widehat{\tau_{c,T}} \qquad (14)$$

Therefore, private consumption increases less than global public expenditure and the budgetary multiplier is smaller than one if the consumption taxation rate increases. However, the rest of the model will allow to distinguish between the various components of this budgetary expenditure.

3.2 Firms

The representative firm produces a differentiated good in a monopolistic competition setting. It defines prices in order to maximize its profit, taking other variables as given. The firm rents capital and labor on perfectly competitive markets. Capital is defined according to equation (5), whereas labor supply is defined according to the maximization program of households in equation (13). Monopolistic competition gives to goods suppliers a market power regarding price-setting, while at the same time fitting the empirical evidence of a large number of firms in the economy. So, the production function of the representative firm, including the utilization of capital and labor, has the following form:

$$Y_t = A_t K_t^{\nu} L_t^{1-\nu} G_{inv,t}^{z_1} G_{c,t}^{z_2} \qquad 0 < \nu < 1 \qquad 0 < z_2 < z_1 < 1 \quad (15)$$

With (Y_t): real production level; (A_t): technology or productivity shock, evolving exogenously over time; (ν): share of capital in the production function; (z_1) or (z_2): productivity of public expenditure.

Finn (1998), Straub and Tchakarov (2007), Pappa (2009), Leeper *et al.* (2010) or Bhattarai and Trzeciakiewicz (2017) introduce public capital instead of public expenditure in the production function of the representative firm[7]. Carvalho and Martins (2011) consider low productive, highly productive public spending and public employment. In the same way, Finn (1998) distinguishes between investment (public capital expenditure) and public employment spending. In this paper, we have chosen to introduce the fact that public expenditure is made freely available by the government, and can be more (z_1 is high) or less (z_2 is small) efficient and productive in increasing the productivity of private factors. So, investment public expenditure ($G_{inv,t}$) is supposed to be more productive than consumption public spending ($G_{c,t}$), which implies ($z_1>z_2$), whereas: ($G_t=G_{inv,t}+G_{c,t}$).

The firm maximizes its nominal profit: $\Xi_t = P_t Y_t - W_t L_t - R_t^k K_t$. So, this implies[8]:

$$\frac{R_t^k K_t}{W_t L_t} = \frac{\nu}{(1-\nu)} \qquad (16)$$

The capital-labor ratio is thus defined by the returns in the production function. So, in logarithms and in variations, we have:

$$k_t - l_t + r_t^k - w_t = 0 \qquad (17)$$

Therefore, by combining equations (15) and (17), we can obtain:

$$\nu\left(w_t - r_t^k\right) = y_t - a_t - l_t - z_1 g_{inv,t} - z_2 g_{c,t} \qquad (18)$$

Let's consider a Calvo-type framework of staggered priced, where a fraction ($0<\alpha<1$) of goods prices remain unchanged each period, whereas prices are adjusted for the remaining fraction ($1-\alpha$) of goods. Monopolistically competitive firms choose their nominal prices to maximize profits subject to constraints on the frequency of future price adjustments, and taking into account that prices may be fixed for many periods. So, they minimize the loss function:

$$Min_{p_t^r} \sum_{k=0}^{\infty} (\alpha\beta)^k E_t (p_t^r - \widetilde{p_{t+k}^r})^2 \qquad (19)$$

[6] $\frac{\partial U_T}{\partial G_T} = \alpha_g (G_T)^{-\frac{1}{\theta}} = \frac{1}{\tau_{c,T}} \frac{\partial U_T}{\partial C_T} = \frac{\alpha_c}{\tau_{c,T}} (C_T)^{-\frac{1}{\theta}}$.

[7] Leeper *et al.* (2010) make the distinction between *authorized* public expenditure and *implemented* public expenditure, as implementation lags can increase the gap between authorization of funding and project completion. Nevertheless, we will not take into account this distinction in the current paper.

[8] The maximization of the profit implies: $\frac{\partial \Xi_t}{\partial K_t} = \nu P_t A_t K_t^{\nu-1} L_t^{1-\nu} G_{inv,t}^{z_1} G_{c,t}^{z_2} - R_t^k = 0$; $\frac{\partial \Xi_t}{\partial L_t} = (1-\nu) P_t A_t K_t^{\nu} L_t^{-\nu} G_{inv,t}^{z_1} G_{c,t}^{z_2} - W_t = 0$.

Where $(\widetilde{p_t^r})$ is the logarithm of the optimal price that the representative firm will set in period (t) if there were no price rigidity.

The firm minimizes expected losses in profit for all future periods (t+k) due to the fact that it will not be able to set a frictionless optimal price in this period (t+k).These losses are subject to the actualization rate (β), as closer profits are given a higher weight than more distant ones. Besides, the probability that the price (p_t^r) will be fixed for (k) periods, until the period (t+k), is (α^k). Thus, by deriving in function of the reset price (p_t^r), we have:

$\sum_{k=0}^{\infty}(\alpha\beta)^k\, p_t^r = \frac{1}{(1-\alpha\beta)} p_t^r = \sum_{k=0}^{\infty}(\alpha\beta)^k E_t(\widetilde{p_{t+k}^r})$, which implies:

$$p_t^r = (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k E_t(\widetilde{p_{t+k}^r}) \qquad (20)$$

Therefore, the representative firm sets the optimal reset price (p_t^r) to the level of a weighted average of the prices that it would have expected to reset in the future if there weren't any price rigidities.

The optimal strategy of the firm is to fix prices at marginal costs: $(\widetilde{\mathrm{p}_{\mathrm{t}}^{\mathrm{r}}} = mc_t)$, where (mc_t) is the nominal marginal production cost of the representative firm. Furthermore, prices in period (t) are an average of past prices and reset prices:

$$p_t = \alpha p_{t-1} + (1-\alpha)p_t^r \qquad (21)$$

So, by combining equations (20) and (21)[9], we obtain:

$$\mathrm{p}_{\mathrm{t}}^{\mathrm{r}} = \frac{1}{(1-\alpha)}p_t - \frac{\alpha}{(1-\alpha)}p_{t-1} = \frac{\alpha\beta}{(1-\alpha)}\mathrm{E}_{\mathrm{t}}(p_{t+1}) - \frac{\alpha^2\beta}{(1-\alpha)}p_t + (1-\alpha\beta)mc_t \quad (22)$$

Therefore, we have the following inflation rate:

$$\pi_t = p_t - p_{t-1} = \beta E_t(\pi_{t+1}) + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(mc_t - p_t) \quad (23)$$

Inflation then depends on expected future inflation, and on the gap between the frictionless optimal price level and the current price level, i.e.: on the real marginal cost. Indeed, inflationary pressures can be due to the fact that prices which can be reset by firms are increased.

We still have to clarify the expression of the real marginal production cost for the representative firm. According to equation (15), the production costs of the quantity (Y_t) are:

$W_tL_t = W_t\left(\frac{Y_t}{A_tK_t^{\nu}G_{inv,t}^{z_1}G_{c,t}^{z_2}}\right)^{\frac{1}{1-\nu}}$ and $R_t^kK_t = R_t^k\left(\frac{Y_t}{A_tL_t^{1-\nu}G_{inv,t}^{z_1}G_{c,t}^{z_2}}\right)^{\frac{1}{\nu}}$

So, differentiating these expressions, and using equations (15) and (16), the nominal marginal production cost of the quantity (Y_t) is:

$$MC_t = \frac{\partial(W_tL_t)}{\partial Y_t} = \frac{W_tL_t^{\nu}}{(1-\nu)A_tK_t^{\nu}G_{inv,t}^{z_1}G_{c,t}^{z_2}}$$

$$= \frac{\partial(R_t^kK_t)}{\partial Y_t} = \frac{R_t^kK_t^{1-\nu}}{\nu A_t{L_t}^{1-\nu}G_{inv,t}^{z_1}G_{c,t}^{z_2}} = \frac{(R_t^k)^{\nu}W_t^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}A_tG_{inv,t}^{z_1}G_{c,t}^{z_2}} \quad (24)$$

In logarithms, we obtain the following variation in the real marginal production cost:

$$(mc_t - p_t) = (1-\nu)(w_t - p_t) + \nu\left(r_t^k - p_t\right) - a_t - z_1 g_{inv,t} - z_2 g_{c,t} \qquad (25)$$

So, obviously, real marginal production costs increase with the real wage and with the real cost of capital services, whereas they decrease with productivity and with public expenditure. Therefore, a public spending shock increases global demand, labor demand and real wages, as well as marginal production costs, which is detrimental to economic growth and productivity; unless the productivity of public spending (z) is sufficient to compensate for the former effect and to reduce real production costs. Anyway, real marginal production costs increase less in case of productive public spending (z_1 is higher). The inflationary tensions are then less accentuated, and the monetary authority increases less the nominal interest rate, which is less detrimental to private consumption and to economic growth.

---

[9] Equation (20) implies: $\mathrm{p}_t^{\mathrm{r}} = \alpha\beta\mathrm{E}_{\mathrm{t}}(\mathrm{p}_{\mathrm{t+1}}^{\mathrm{r}}) + (1-\alpha\beta)\widetilde{\mathrm{p}_{\mathrm{t}}^{\mathrm{r}}}$; eq. (21) implies: $\mathrm{p}_{\mathrm{t}}^{\mathrm{r}} = \frac{1}{(1-\alpha)}p_t - \frac{\alpha}{(1-\alpha)}p_{t-1}$.

Besides, for a given period (T), equations (13) and (18) imply:

$$(1+\nu\varphi)(w_T-p_T) = \varphi y_T - \varphi(a_T + z_1 g_{inv,T} + z_2 g_{c,T}) + \nu\varphi\left(r_T^k - p_T\right) + \left(\widehat{\tau_{l,T}} + \widehat{\tau_{c,T}}\right) + \frac{1}{\theta}c_T \quad (26)$$

Therefore, equations (7), (23), (25) and (26) imply the following inflation rate, for a given period (T):

$$\pi_T = \beta E_t(\pi_{T+1}) + \frac{\varphi k_1(1-\nu)}{(1+\nu\varphi)}y_T + \frac{k_1(1-\nu)}{\theta(1+\nu\varphi)}c_T - \frac{k_1(1+\varphi)}{(1+\nu\varphi)}\left(a_T + z_1 g_{inv,T} + z_2 g_{c,T}\right) + \frac{k_1(1-\nu)}{(1+\nu\varphi)}\left(\widehat{\tau_{l,T}} + \widehat{\tau_{c,T}}\right) + \frac{\nu k_1(1+\varphi)}{(1+\nu\varphi)}\widehat{\tau_{k,T}} \quad (27)$$

$$with: k_1 = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$$

Indeed, real marginal production costs and prices increase with private economic activity (c_t, y_t), because of the expansionary effect of economic activity on labor demand and employment. They also increase with taxation rates ($\tau_{l,t}$, $\tau_{c,t}$, $\tau_{k,t}$). Obviously, real marginal production costs and prices also decrease with positive productive technology shocks (a_t).

3.3 Global equilibrium

We are now going to derive the equilibrium on the goods market regarding the global demand. Clearing on the goods market in period (T) requires:

$$Y_T = C_T + G_T + INV_T \quad (28)$$

Therefore, in logarithms and in variations, we obtain:

$$y_T = \frac{C_T}{Y_T}c_T + \frac{G_T}{Y_T}g_T + \frac{INV_T}{Y_T}inv_T \quad (29)$$

Besides, profit maximization in the footnote 6 and equations (5) and (7) imply:

$$\frac{INV_T}{Y_T} = \left(\frac{INV_T}{K_T}\right)\left(\frac{K_T}{Y_T}\right) = \delta\frac{\nu P_T}{R_T^k} = \frac{\delta\nu\beta(1-\tau_{k,T})}{[(1-\beta)+\delta\beta(1-\tau_{k,T})]} < 1$$

$$(inv_T - y_T) = -\left(r_T^k - p_T\right) = -\widehat{\tau_{k,T}} \quad (30)$$

Let's define the output-gap as the differential between effective and potential output ($x_T = y_T - y_T^p$). By combining equations (11), (14), (28), (29) and (30)[10], we obtain:

$$y_T = E_T(y_{T+1}) - \theta[i_T - E_T(\pi_{T+1})] - \theta\left\{1 - \left(\frac{G_T}{Y_T}\right)\frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]}\right\}\widehat{\tau_{c,T}} - \frac{\delta\nu\beta(1-\tau_{k,T})}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]}\widehat{\tau_{k,T}} + \frac{\delta\nu\beta(1-\tau_{k,T+1})}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T+1})]}E_T(\widehat{\tau_{k,T+1}}) + \theta\left\{1 - E_T\left(\frac{G_{T+1}}{Y_{T+1}}\right)\frac{\theta[(1-\beta)+\delta\beta(1-\tau_{k,T+1})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T+1})]}\right\}E_T\left(\widehat{\tau_{c,T+1}}\right) \quad (31)$$

Therefore, using the definition of potential output, this equation implies:

$$x_T = E_T(x_{T+1}) - \theta[i_T - E_T(\pi_{T+1}) - \overline{r_T}] \quad (32)$$

$$\overline{r_T} = -\frac{1}{\theta}[y_T^p - E_T(y_{T+1}^p)] - \left\{1 - \left(\frac{G_T}{Y_T}\right)\frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]}\right\}\widehat{\tau_{c,T}} - \frac{\delta\nu\beta(1-\tau_{k,T})}{\theta[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]}\widehat{\tau_{k,T}} + \frac{\delta\nu\beta(1-\tau_{k,T+1})}{\theta[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T+1})]}E_T(\widehat{\tau_{k,T+1}})$$

---

[10] (11) implies: $c_T = E_T(c_{T+1}) - \theta[i_T - E_T(\pi_{T+1})] - \theta[\widehat{\tau_{c,T}} - E_T(\widehat{\tau_{c,T+1}})]$.
(14), (28), (29) and (30) imply: $y_T = c_T + \frac{\theta G_T}{(Y_T - INV_T)}\widehat{\tau_{c,T}} - \frac{INV_T}{(Y_T - INV_T)}\widehat{\tau_{k,T}}$
$y_T = c_T + \left(\frac{G_T}{Y_T}\right)\frac{\theta[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]}\widehat{\tau_{c,T}} - \frac{\delta\nu\beta(1-\tau_{k,T})}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]}\widehat{\tau_{k,T}}$.

$$+\left\{1-E_T\left(\frac{G_{T+1}}{Y_{T+1}}\right)\frac{\theta\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T+1}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T+1}\right)\right]}\right\}E_T\left(\widehat{\tau_{c,T+1}}\right)$$

- (θ): real interest rate elasticity of the output-gap, 'inter-temporal elasticity of substitution' of household expenditure.
- $\overline{r_T}$: Equilibrium or natural real interest rate, which corresponds to the steady-state real rate of return if prices and wages were fully flexible. It is the real interest rate required to keep aggregate demand equal at all times to the natural rate of output. It includes all non-monetary disturbances. It is a decreasing function of the temporary increase in potential output or in consumption or capital taxation rates.

So, according to equation (32), higher future expected output increases current output and consumption, because households prefer to smooth consumption, and then higher future revenues raise their current consumption and current output. Current output is also a decreasing function of the excess of the real interest rate above its natural level, because of the inter-temporal substitution of consumption.

So, we obtain the following components of global demand:

$$y_T = x_T + y_T^p \quad (33)$$

Equation (30) implies: $inv_T = y_T - \widehat{\tau_{k,T}} = x_T + y_T^p - \widehat{\tau_{k,T}}$ (34)

Equation (14) and footnote 9 imply:

$$c_T = g_T - \theta\widehat{\tau_{c,T}} = x_T + y_T^p - \left(\frac{G_T}{Y_T}\right)\frac{\theta\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{c,T}} + \frac{\delta\nu\beta\left(1-\tau_{k,T}\right)}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \quad (35)$$

$g_{inv,T} = x_T + y_T^p + \varepsilon_T^{g,inv}$ (36) if we note ($\varepsilon_T^{g,inv}$) the shock on public investment

$$g_{c,T} = \frac{G_T}{G_{c,T}}g_T - \frac{G_{inv,T}}{G_{c,T}}g_{inv,T} = x_T + y_T^p - \frac{\left(G_{c,T}+G_{inv,T}\right)^2}{G_{c,T}Y_T}\frac{\theta\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{c,T}} + \theta\left(\frac{G_T}{G_{c,T}}\right)\widehat{\tau_{c,T}} + \frac{G_T}{G_{c,T}}\frac{\delta\nu\beta\left(1-\tau_{k,T}\right)}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} - \frac{G_{inv,T}}{G_{c,T}}\varepsilon_T^{g,inv} \quad (37)$$

Besides, using equations (27), (35), (36) and (37), the supply function is as follows, in differential with the long run equilibrium value:

$$\pi_T = \beta E_t(\pi_{T+1}) + k_1k_2x_T \quad (38)$$

$$with:\ k_2 = \frac{[(1-\nu)(1+\varphi\theta)-\theta(1+\varphi)(z_1+z_2)]}{(1+\nu\varphi)\theta}$$

$$y_T^p = \frac{(1+\varphi)}{k_2(1+\nu\varphi)}a_T + \frac{(1+\varphi)}{k_2(1+\nu\varphi)}\left(z_1 - z_2\frac{G_{inv,T}}{G_{c,T}}\right)\varepsilon_T^{g,inv} - \frac{(1-\nu)}{k_2(1+\nu\varphi)}\widehat{\tau_{l,T}} - \frac{\left[(1-\nu)G_{c,T}-\theta(1+\varphi)z_2G_T\right]}{(1+\nu\varphi)k_2G_{c,T}}\left\{1-\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} - \frac{\nu(1+\varphi)}{k_2(1+\nu\varphi)}\widehat{\tau_{k,T}} + \frac{\delta\nu\beta\left(1-\tau_{k,T}\right)\left[\theta(1+\varphi)z_2G_T-(1-\nu)G_{c,T}\right]}{\theta k_2(1+\nu\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]G_{c,T}}\widehat{\tau_{k,T}}$$

Equation (38) implies:

$$\left(z_1 - z_2\frac{G_{inv,T}}{G_{c,T}}\right)\varepsilon_T^{g,inv} = \frac{k_2(1+\nu\varphi)}{(1+\varphi)}y_T^p - a_T + \frac{(1-\nu)}{(1+\varphi)}\widehat{\tau_{l,T}} + \frac{\left[(1-\nu)G_{c,T}-\theta(1+\varphi)z_2G_T\right]}{(1+\varphi)G_{c,T}}\left\{1-\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} + \nu\widehat{\tau_{k,T}} - \frac{\delta\nu\beta\left(1-\tau_{k,T}\right)\left[\theta(1+\varphi)z_2G_T-(1-\nu)G_{c,T}\right]}{\theta(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]G_{c,T}}\widehat{\tau_{k,T}} \quad (B2)$$

- (y_t^p): potential output: it is an increasing function of positive productivity shocks (a_t), and of positive shocks on public investment ($\varepsilon_t^{g,inv}$). However, potential output is negatively correlated with taxation rates: on labor, capital or consumption.

Equation (38) is the simplest form of the New-Keynesian Phillips curve. In this equation, ($k_1 k_2$) is an indicator of price flexibility. This parameter decreases with the indicator of price-stickiness (α), the longer the average time interval between price changes. It increases with the share of labor in the production function (1-ν), and with the impatience of households to consume (it decreases with β). Price flexibility increases with the dis-utility, for households, of labor supply (φ), as labor supply is then more elastic to the level of the real wage. However, it also decreases with the inter-temporal elasticity of substitution of household expenditure (θ), and with the productivity of public expenditure (z_1 and z_2).

3.4 Monetary policy

The interest rate reacts to inflation and economic activity deviations according to a simple Taylor rule, but we also introduce a high degree of interest rate smoothing. So, the nominal interest rate is fixed by the common central bank as follows:

$$i_T = \lambda_{i,CB} i_{T-1} + \lambda_{\pi,CB}(\pi_T - \pi^{opt}) + \lambda_{x,CB}(x_T - x^{opt}) \qquad (39)$$

where ($\lambda_{i,CB}$), ($\lambda_{\pi,CB}$) and ($\lambda_{x,CB}$) are the respective weights given by the central bank to interest rate smoothing, stabilizing prices and the output-gap.

Therefore, with equations (C1) and (C2) in Appendix (C), we obtain:

$$\begin{aligned} i_T =& \frac{(1-\delta)k_2}{\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB}k_1k_2\right)\theta\right]}\left[\lambda_{i,CB} i_{T-1} - \lambda_{\pi,CB}\pi^{opt} - \lambda_{x,CB}x^{opt}\right] \\ &+ \frac{\left(\lambda_{x,CB} + \lambda_{\pi,CB}k_1k_2\right)\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB}k_1k_2\right)\theta\right]}\widehat{\tau_{k,T}} \\ &+ \frac{\theta\left(\lambda_{x,CB} + k_1k_2\lambda_{\pi,CB}\right)}{\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB}k_1k_2\right)\theta\right]}\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} \\ &+ f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty} \tau_{c,n}, \sum_{n=T+1}^{\infty} \tau_{k,n}\right) \qquad (40) \end{aligned}$$

3.5 Budgetary policy

In the economic literature, public expenditure is often modelled as following an AR(1) process:

$$g_t = \rho_{g,g} g_{t-1} - \rho_{gc,x} x_{t-1} + \varepsilon_t^g \qquad (41) \qquad 0 < \rho_{g,g} < 1 \quad 0 < \rho_{gc,x} < 1 \qquad \varepsilon_t^g \sim N\left(0, \sigma_g^2\right)$$

In this framework, in order to introduce the counter-cyclical role of budgetary policy, public consumption expenditure can be supposed to decrease with the level of the output gap. Indeed, transfers in the form of unemployment and welfare benefits decrease in case of economic growth ($\rho_{gc,x}$>0). However, the latter coefficient seems empirically quite small [for example, ($\rho_{gc,x} = 0.05$) in Zubairy (2014)]. On the opposite, in Sims and Wolff (2013), in Gali *et al.* (2007) or in Straub and Tchakarov (2007), the auto-correlation of public expenditure is really high: ($\rho_{g,g} = 0.9$). In Coenen and Straub (2005), it is ($\rho_{g,g} = 0.85$), it is ($\rho_{g,g} = 0.83$) in Bouakez and Rebei (2007), whereas it is ($\rho_{g,g} = 0.8$) in Zubairy (2014), and it is only ($\rho_{g,g} = 0.7$) in Leeper *et al.* (2011).

On the contrary, in the current paper, the optimal variation in public expenditure depends on a weighted average of variations in taxation rates, on the technological progress, and on the monetary policy (nominal interest rate) conducted by the central bank. Indeed, according to equations (B11) and (B12) in Appendix B and to equations (C1) and (C3) in Appendix C, we obtain the following optimal levels of public expenditure:

$$g_{inv,T} = -\frac{\left[(1-\nu)(1-\delta-\delta\varphi\theta)G_{c,T}+\theta(1+\varphi)G_{c,T}-\theta(1-\delta)(1+\varphi)z_2G_T\right]}{(1-\delta)\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)}i_T$$
$$-\frac{[(1+\nu\varphi)-\nu(1-\nu)(1-\delta)]\delta\beta\left(1-\tau_{k,T}\right)G_{c,T}}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}}$$
$$-\frac{[1-\nu+\nu\delta(1+\varphi)](1-\beta)G_{c,T}}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}}$$
$$-\frac{G_{c,T}}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)}a_T+\frac{(1-\nu)G_{c,T}}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)}\widehat{\tau_{l,T}}$$
$$-\frac{\theta[(1+\varphi)-\delta\varphi(1-\nu)]G_{c,T}}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1-\delta)}\left\{1-\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}}$$
$$+f\left(\sum_{n=T+1}^{\infty}i_n\,,\sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}\,,\sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right)\quad(41)$$

$$g_{c,T} = \frac{\left[(1-\nu)(1-\delta-\delta\varphi\theta)G_{inv,T}+\theta(1+\varphi)G_{inv,T}-\theta(1-\delta)(1+\varphi)z_1G_T\right]}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)(1-\delta)}i_T$$
$$+\frac{[(1+\nu\varphi)-\nu(1-\nu)(1-\delta)]\delta\beta\left(1-\tau_{k,T}\right)G_{inv,T}}{(1+\varphi)(1-\delta)\left(z_1G_{c,T}-z_2G_{inv,T}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}}$$
$$+\frac{[1-\nu+\nu\delta(1+\varphi)](1-\beta)G_{inv,T}}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}}$$
$$+\frac{G_{inv,T}}{(z_1G_{c,T}-z_2G_{inv,T})}a_T-\frac{(1-\nu)G_{inv,T}}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)}\widehat{\tau_{l,T}}$$
$$+\frac{\theta[(1+\varphi)-\delta\varphi(1-\nu)]G_{inv,T}}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1-\delta)}\left\{1-\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}}$$
$$+f\left(\sum_{n=T+1}^{\infty}i_n\,,\sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}\,,\sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right)(42)$$

Therefore, with our basic calibration ($z_1G_{c,T} > z_2G_{inv,T}$), a higher level of technological progress increases public consumption expenditure and strongly reduces public investment expenditure. We will study the consequences of variations in taxation rates in the following section 4.

Besides, we will also introduce in section 5 the fact that the budgetary policy can be active. Indeed, instead of simply conducting the optimal previous budgetary policy, the budgetary authority can be constrained by the public indebtedness level and by the necessity to tend towards the budgetary balance. So, the public debt increases with the weight of the reimbursement of the previous public debt level, and with investment and consumption public expenditure to be financed. It decreases with taxes collected: on labor, consumption and capital. Therefore, we have the following expression of the public debt constraint:

$$B_t = (1+i_{t-1})B_{t-1}+P_tG_{inv,t}+P_tG_{c,t}-\tau_{l,t}W_tL_t-\tau_{c,t}P_tC_t-\tau_{k,t}\left(R_t^k-\delta P_t\right)K_t\quad(43)$$

With: (B_t): stock of nominal public debt at the end of period (t) and at the beginning of period (t+1). Public expenditures appear in the budgetary constraint whatever their productivity. Besides, in equilibrium on the capital market, the nominal interest rate on the public debt is the same as the market interest rate (i_t).

In section 5 of the paper, in a very simple way, we will model the budgetary constraint and the necessity to avoid an outbidding of the public debt by the fact that fiscal resources must increase in proportion to public expenditure. Therefore, the footnote 7 and equations (7) imply:

$$(1-\nu)\widehat{\tau_{l,t}} = -\frac{(1-\beta)\nu}{\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)}\widehat{\tau_{k,t}} - \frac{C_t}{Y_t}\widehat{\tau_{c,t}} \qquad (44)$$

So, if we suppose that the government tries to balance the budget with the labor taxation rate, if the budgetary policy is active, according to equations (41), (42) and (44), we have the following public expenditures:

$$\begin{aligned} g_{inv,T} = & -\frac{G_{c,T}}{(1+\varphi)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)}\left\{\frac{\theta[(1+\varphi)-\delta\varphi(1-\nu)]}{(1-\delta)} + \left(\frac{C_t}{Y_t}\right)\right. \\ & \left. - \frac{\theta[(1+\varphi)-\delta\varphi(1-\nu)]\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} \\ & -\frac{\left[(1-\nu)(1-\delta-\delta\varphi\theta)G_{c,T} + \theta(1+\varphi)G_{c,T} - \theta(1-\delta)(1+\varphi)z_2 G_T\right]}{(1-\delta)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)} i_T \\ & -\frac{\{\delta\beta[1+\nu\varphi-\nu(1-\nu)(1-\delta)]\left(1-\tau_{k,t}\right) + (1+\nu\varphi)(1+\nu\delta)(1-\beta)\}\delta\beta\left(1-\tau_{k,T}\right)G_{c,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \\ & -\frac{(1-\beta)(1+\nu\delta\varphi)G_{c,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)(1+\varphi)(1-\delta)}\widehat{\tau_{k,T}} \\ & -\frac{G_{c,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)} a_T + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty} \widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty} \widehat{\tau_{k,n}}\right) \end{aligned} \qquad (45)$$

$$\begin{aligned} g_{c,T} = & \frac{G_{inv,T}}{(1+\varphi)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)}\left\{\frac{\theta[(1+\varphi)-\delta\varphi(1-\nu)]}{(1-\delta)} + \left(\frac{C_T}{Y_T}\right)\right. \\ & \left. - \frac{\theta[(1+\varphi)-\delta\varphi(1-\nu)]\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} \\ & +\frac{\left[(1-\nu)(1-\delta-\delta\varphi\theta)G_{inv,T} + \theta(1+\varphi)G_{inv,T} - \theta(1-\delta)(1+\varphi)z_1 G_T\right]}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)(1-\delta)} i_T \\ & +\frac{\{\delta\beta\left(1-\tau_{k,t}\right)[1+\nu\varphi-\nu(1-\nu)(1-\delta))] + (1-\beta)(1+\nu\varphi)(1+\nu\delta)\}\delta\beta\left(1-\tau_{k,T}\right)G_{inv,T}}{(1+\varphi)(1-\delta)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \\ & +\frac{(1+\nu\delta\varphi)(1-\beta)G_{inv,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)(1-\delta)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)}\widehat{\tau_{k,T}} \\ & +\frac{G_{inv,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)} a_T + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty} \widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty} \widehat{\tau_{k,n}}\right) \end{aligned} \qquad (46)$$

3.6 Calibration

The preference for the present (β) is usually calibrated at (0.99).

The intertemporal elasticity of substitution (θ) is supposed to be (0.5) in Leeper *et al.* (2011). It is (0.57) in Leeper *et al.* (2010), (0.85) in Drautzberg and Uhlig (2011), (0.66) in Sims and Wolff (2013), It is assumed to be (1) in Galí *et al.* (2007), in Coenen and Straub (2005) or in Smets and Wouters (2003), where consumption appears in logarithm in the utility function (consistent with log preferences). In this paper, we will consider (θ=1).

The inverse of the Frisch elasticity of labor supply (φ) is only (0.2) in Gali *et al.* (2007). It is supposed to be (1) in Zubairy (2014), or in Sims and Wolff (2013). This elasticity is (2) in Coenen and Straub (2005), in Smets and Wouters (2003), in Leeper *et al.* (2010, 2011); it is even (2.16) in Drautzberg and Uhlig (2011). In this paper, we will consider (φ=1).

The depreciation rate of capital is supposed to be (δ=0.025) in Zubairy (2014), in Sims and Wolff (2013), in Gali *et al.* (2007), in Finn (1998), in Smets and Wouters (2003), in Leeper *et al.* (2010) or in Mertens and Ravn (2011). It is only weaker in Drautzberg and Uhlig (2011), where (δ=0.0145). So, in this paper, we will retain the following value: (δ=0.025).

The share of capital in the production function (elasticity of output to the capital stock) is supposed to be (ν=0.24) in Drautzberg and Uhlig (2011). It is (ν=0.30) in Smets and Wouters (2003) or in Finn (1998). Straub and Tchakarov (2007) calibrate the labor share of output at (1-ν=0.7), the private capital share of output at (ν=0.285) and the public capital share at (ν=0.015). This share is calibrated at (ν=0.33) in Sims and Wolff (2013) or in Gali *et al.* (2007). It is (ν=0.36) in Leeper *et al.* (2010), in Mertens and Ravn (2011) or in Bouakez and Rebei (2007). So, in this paper, we will consider (ν=0.3).

Inertia in prices remaining unchanged (α) is supposed to be (0.5) in Sims and Wolff (2013). It is (0.75) in Gali *et al.* (2007) or in Coenen and Straub (2005), whereas it is (0.81) in Drautzberg and Uhlig (2011). In this paper, we will consider (α=0.75).

Finn (1998) considers that for public capital investment (highly productive): (z_1=0.16). The productivity of public consumption expenditure is supposed to be (z_2=0.05) and the productivity of investment expenditure is supposed to be (z_1=0.2) in Carvalho and Martins (2011). In this paper, we will consider these former values.

The respective shares of demand components in output are supposed to be: ($\frac{C}{Y}=0.6$), ($\frac{INV}{Y}=0.22$)[11] and ($\frac{G}{Y}=0.18$) in Smets and Wouters (2003). The share of government spending in output is: ($\frac{G}{Y}=0.18$) in Zubairy (2014), ($\frac{G}{Y}=0.2$) in Sims and Wolff (2013), in Mertens and Ravn (2011) or in Gali *et al.* (2007). In Leeper *et al.* (2010), the shares of public expenditure in GDP are: ($\frac{G_c}{Y}=0.144$) and ($\frac{G_{inv}}{Y}=0.038$). In Drautzberg and Uhlig (2011), they are: ($\frac{G_c}{Y}=0.153$) and ($\frac{G_{inv}}{Y}=0.04$). Following the NAWM (New Area Wide Model), Straub and Tchakarov (2007) calibrate the ratios of public consumption to output at (0.155) in the European Union and (0.128) in the United States, and the ratios of public investment to output at (0.025) and (0.032) respectively. Private consumption to GDP is (0.6) in the European Union and (0.62) in the United States, whereas private investment to GDP is (0.22) in both regions. In Bhattarai and Trzeciakiewicz (2017), these shares are ($\frac{C}{Y}=0.63$), ($\frac{INV}{Y}=0.15$), ($\frac{G_c}{Y}=0.20$), ($\frac{G_{inv}}{Y}=0.02$). So, in the paper, we will consider: ($\frac{C}{Y}=0.64$), ($\frac{G_c}{Y}=0.15$), ($\frac{G_{inv}}{Y}=0.03$).

The steady–state capital income tax rate is supposed to be ($\tau_k=0.19$) in Forni *et al.* (2009), ($\tau_k=0.36$) in Drautzberg and Uhlig (2011), ($\tau_k=0.384$) in Leeper *et al.* (2010), ($\tau_k=0.4071$) in Bhattarai and Trzeciakiewicz (2017), ($\tau_k=0.41$) in Zubairy (2014), ($\tau_k=0.42$) in Mertens and Ravn (2011), and ($\tau_k=0.43$) in Finn (1998). The labor income tax rate is supposed to be ($\tau_l=0.20$) in Carvalho and Martins (2011), ($\tau_l=0.214$) in Leeper *et al.* (2010), ($\tau_l=0.23$) in Zubairy (2014), ($\tau_l=0.25$) in Finn (1998), ($\tau_l=0.26$) in Mertens and Ravn (2011), ($\tau_l=0.28$) in Drautzberg and Uhlig (2011), and ($\tau_l=0.2844$) in Bhattarai and Trzeciakiewicz (2017). The consumption tax rate is supposed to be ($\tau_c=0.05$) in Drautzberg and Uhlig (2011), ($\tau_c=0.095$) in Leeper *et al.* (2010), ($\tau_c=0.16$) in Forni *et al.* (2009), ($\tau_c=0.20$) in Coenen and Straub (2005) or in Carvalho and Martins (2011), and ($\tau_c=0.2008$) in Bhattarai and Trzeciakiewicz (2017). In this paper, we will consider: ($\tau_k=0.4$), ($\tau_l=0.22$), ($\tau_c=0.20$).

Regarding monetary policy, we consider that the parameters vary according to the empirical policy preferences and objectives (disinflation in the 1980's, fight against deflationary tensions today), and to the high degree of interest rate smoothing observed in the empirical behavior of central banks. Zubairy (2014) takes: ($\lambda_{i,CB}=0.8$), ($\lambda_{\pi,CB}=0.32$) and ($\lambda_{x,CB}=0.02$). Sims and Wolff (2013) take: ($\lambda_{i,CB}=0.9$), ($\lambda_{\pi,CB}=0.15$) and ($\lambda_{x,CB}=0.01$). In Coenen and Straub (2005), the parameters are: ($\lambda_{i,CB}=0.8$), ($\lambda_{\pi,CB}=0.34$), ($\lambda_{x,CB}=0.02$). In Leeper *et al.* (2011), they are: ($\lambda_{i,CB}=0.7$), ($\lambda_{\pi,CB}=0.45$), ($\lambda_{x,CB}=0.05$). In, Forni *et al.* (2009), they are: ($\lambda_{i,CB}=0.9$), ($\lambda_{\pi,CB}=0.18$), ($\lambda_{x,CB}=0.01$). Finally, in Drautzberg and Uhlig (2011), the parameters are: ($\lambda_{i,CB}=0.92$), ($\lambda_{\pi,CB}=0.13$), ($\lambda_{x,CB}=0.01$). In this paper, we will consider: ($\lambda_{i,CB}=0.8$), ($\lambda_{\pi,CB}=0.3$) and ($\lambda_{x,CB}=0.02$).

---

[11] $\frac{INV}{Y}=0.18=\frac{\delta\nu\beta(1-\tau_{k,T})}{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}$ in eq. (30), which is then consistent with a capital taxation rate ($\tau_{k,T}=0.4$).

4 When the budgetary policy is passive

In this section, we consider that the budgetary authority adjusts optimal variations in public expenditure to variations in taxation rates according to equations (41) and (42), without taking into account the public debt. Then, the economic literature has often underlined that an expansionary budgetary policy and decreasing taxation rates could be efficient to sustain economic growth. However, in the framework of our simple DSGE model, which could be the best fiscal policy?

4.1 Variation in the consumption tax rate

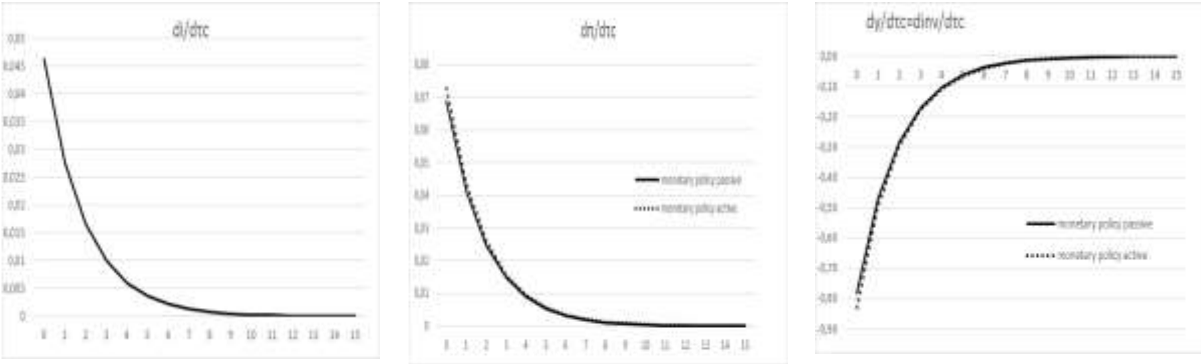
First, what could be the consequences of a decrease in the consumption taxation rate? Equations (40) and (C2) to (C6) in Appendix C imply:

$$\frac{\partial i_T}{\partial \widehat{\tau_{c,T}}} = -\frac{\left(\lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)}{(1-\delta)k_2}\frac{\partial y_T}{\partial \widehat{\tau_{c,T}}} \qquad (47)$$

$$\frac{\partial y_T}{\partial \widehat{\tau_{c,T}}} = \frac{\partial inv_T}{\partial \widehat{\tau_{c,T}}} = -\frac{\theta(1-\delta)k_2}{\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB}k_1k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \qquad (48)$$

$$\frac{\partial \pi_T}{\partial \widehat{\tau_{c,T}}} = -\frac{k_1}{(1-\delta)}\frac{\partial y_T}{\partial \widehat{\tau_{c,T}}} \qquad (49) \qquad\qquad \frac{\partial k_T}{\partial \widehat{\tau_{c,T}}} = -\frac{\delta}{(1-\delta)}\frac{\partial y_T}{\partial \widehat{\tau_{c,T}}} \qquad (50)$$

So, a decrease in the consumption tax rate would slightly reduce inflationary tensions. However, according to our basic calibration, equation (49) shows that if the consumption tax rate decreases by (-1%), prices would only be reduced by (-0.07%) in the first period. So, the nominal interest would only decrease by (-0.05%) in the first period, according to equation (47). Nevertheless, according to equation (48), the expansionary effect on economic activity would be very significant: global economic activity and private investment would increase by 0.78% in the first period. Indeed, as prices are reduced by the weaker consumption taxation rate, goods are less expansive, which strongly encourages private consumption and investment. Global economic activity would still be more improved (around 0.83%) if monetary policy is active and if the interest rate decreases.

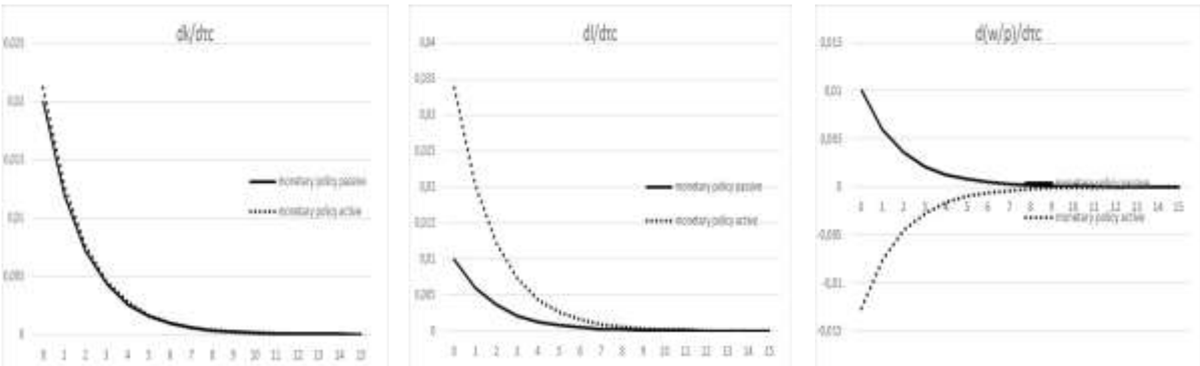


Figures 1: Interest rate, inflation and economic activity after a 1% increase in the consumption taxation rate (persistence: 0.6)

Besides, according to equation (50), if the consumption taxation rate decreases by (-1%) the capital stock very moderately decreases, by (-0.02%) in the first period according to our basic calibration. Furthermore, the decrease in prices improves the purchasing power of households, reduces the working time and the labor supply necessary to reach a given consumption level, and it increases the marginal utility of leisure. Therefore, labor supply slightly decreases by (-0.01%) in the first period (-0.03% if monetary policy is active). However, the consequences on the real wage are much more ambiguous. If the consumption taxation rate decreases by (-1%), the real wage decreases by (-0.01%) in the first period. However, if the monetary policy is active, deflationary tensions are a little bit more accentuated, and therefore, the real wage increases by (+0.01%). Indeed, according to equations (40), (C7) and (C8) in Appendix C, we obtain:

$$\frac{\partial l_T}{\partial \widehat{\tau_{c,T}}} = \frac{\theta\left(\delta k_2 + \lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)}{(1+\varphi)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \quad (51)$$

$$\frac{\partial (w_T - p_T)}{\partial \widehat{\tau_{c,T}}} = \frac{\left(\delta\varphi k_2 - \lambda_{x,CB} - k_1 k_2 \lambda_{\pi,CB}\right)}{\left(\delta k_2 + \lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)} \frac{\partial l_T}{\partial \widehat{\tau_{c,T}}} \quad (52)$$



Figures 2: Capital, labor and real wage after a 1% increase in the consumption taxation rate (persistence: 0.6)

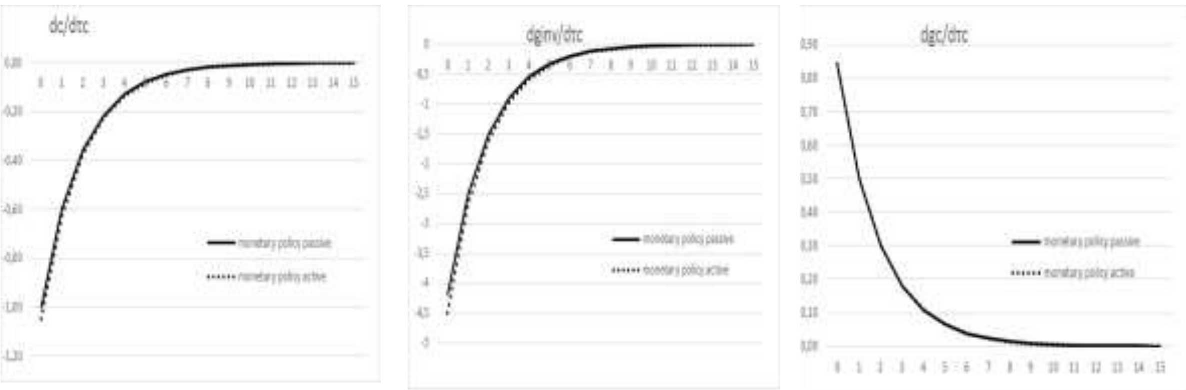
Moreover, the decrease in the consumption tax rate by (-1%) would mostly benefit to private consumption, thanks to the decrease in prices: therefore, private consumption would increase proportionally by 1% (1.05% if monetary policy is active) in the first period. Nevertheless, according to our model, one of the main consequences of a decrease in the consumption taxation rate is also to modify the composition of public expenditure. Indeed, according to the basic calibration of our model, equations (40), (41) and (42) imply:

$$\frac{\partial c_T}{\partial \widehat{\tau_{c,T}}} = \frac{-\theta^2\left(\lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)}{\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} - \theta \quad (53)$$

$$\frac{\partial g_{inv,T}}{\partial \widehat{\tau_{c,T}}} = -\frac{\theta\{(1+\varphi-\delta\varphi+\delta\varphi\nu)k_2 G_{c,T} + \left(\lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)[(1-\nu)G_{c,T} - \theta(1+\varphi)z_2 G_T]\}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \quad (54)$$

$$\frac{\partial g_{c,T}}{\partial \widehat{\tau_{c,T}}} = \frac{\theta\{(1+\varphi-\delta\varphi+\delta\varphi\nu)k_2 G_{inv,T} + \left(\lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)[(1-\nu)G_{inv,T} - \theta(1+\varphi)z_1 G_T]\}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} \quad (55)$$

So, if the consumption taxation rate decreases by (-1%), according to our basic calibration, public investment would increase by 4.18% (4.5% if monetary policy is active), whereas public consumption expenditure would decrease by around (-0.84%). Indeed, in order to satisfy the stronger global demand, economic production must increase, and therefore, public investment expenditure strongly increases in order to improve the productivity of private factors in private firms. This is realized thanks to a re-allocation of public expenditure and to a reduction in public consumption.



Figures 3: Private consumption, public consumption and public investment after a 1% increase in the consumption taxation rate (persistence: 0.6)

Finally, in conclusion, a decrease in the consumption taxation rate would be very efficient in order to increase private economic activity (mainly consumption but also investment), and regarding the public sector, it could also strongly contribute to improve the most productive public investment expenditure.

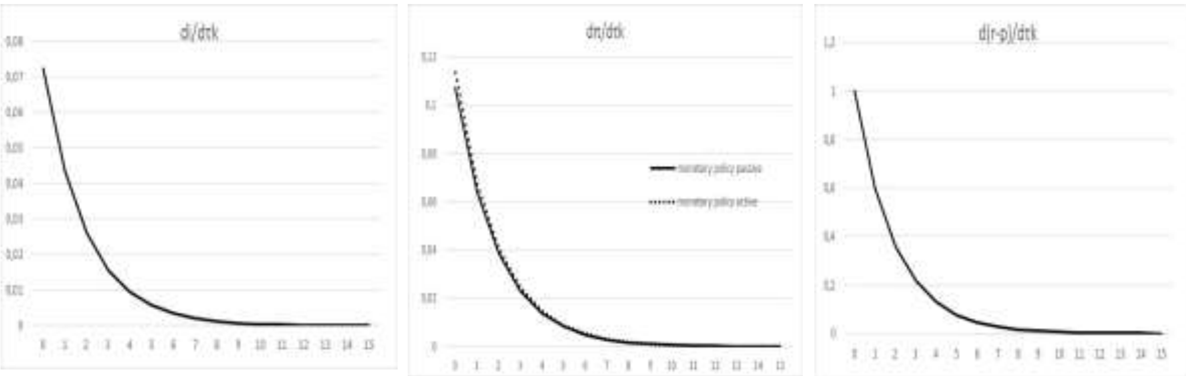
4.2 Variation in the capital tax rate

What could be the consequences of a decrease in the capital taxation rate? Equations (40), (C2) and (C3) in Appendix C imply:

$$\frac{\partial k_T}{\partial \widehat{\tau_{k,T}}} = \frac{\delta k_2\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (56)$$

$$\frac{\partial i_T}{\partial \widehat{\tau_{k,T}}} = \frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)}{\delta k_2}\frac{\partial k_T}{\partial \widehat{\tau_{k,T}}} \quad (57) \qquad \frac{\partial \pi_T}{\partial \widehat{\tau_{k,T}}} = \frac{k_1}{\delta}\frac{\partial k_T}{\partial \widehat{\tau_{k,T}}} \quad (58)$$

So, a decrease in the capital taxation rate would slightly reduce inflationary tensions. However, according to our basic calibration, equation (58) shows that if the capital tax rate decreases by (-1%), prices would only be reduced by (-0.11%) in the first period. So, even if monetary policy is active and more expansionary, the nominal interest would only decrease by (-0.07%) in the first period, according to equation (57).

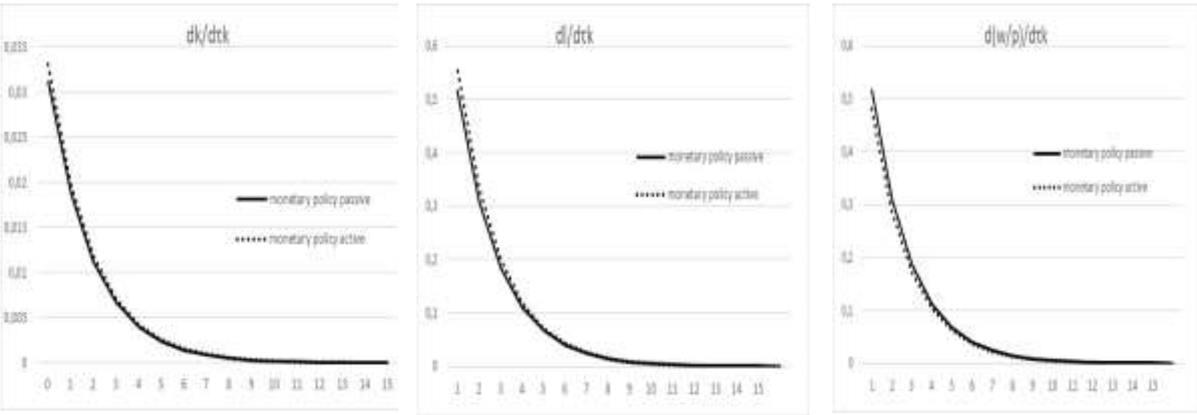


Figures 4: Interest rate, inflation and real capital cost after a 1% increase in the capital taxation rate (persistence: 0.6)

The decrease in the capital taxation rate reduces the real capital cost proportionately. Indeed, equation (7) implies: $\frac{\partial(r_T^k - p_T)}{\partial \tau_{k,T}} = 1$. In this framework, according to equation (56), the decrease in the capital stock would be negligible: if the capital taxation rate decreases by (-1%), it would only be reduced by (-0.03%) in the first period according to our basic calibration. The labor cost and the real wage would also be reduced by around (-0.5%) in the first period, in order to compensate for the relative higher labor marginal cost in comparison with the one of capital. Nevertheless, as the decrease in the capital cost is much stronger, labor demand from firms would be reduced by more than (-0.5%) in the first period; firms substitute capital to labor. Besides, this decrease in labor demand put downward pressure on wages. So, equations (40), (C7) and (C8) in Appendix C imply:

$$\frac{\partial l_T}{\partial \widehat{\tau_{k,T}}} = \frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)(1-\nu+\delta\nu)\right]k_2}{(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} + \frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\left[(1-\beta)(1-\theta)+\delta\beta(1-\theta+\theta\nu)\left(1-\tau_{k,T}\right)\right]}{(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (59)$$

$$\frac{\partial (w_T-p_T)}{\partial \widehat{\tau_{k,T}}} = \frac{\varphi\left[(1-\beta)+\delta\beta(1-\nu+\nu\delta)\left(1-\tau_{k,T}\right)\right]k_2}{(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} - \frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\left[(1-\beta)(1+\varphi\theta)+\delta\beta(1+\varphi\theta-\varphi\theta\nu)\left(1-\tau_{k,T}\right)\right]}{(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (60)$$



Figures 5: Capital, labor and real wage after a 1% increase in the capital taxation rate (persistence: 0.6)

Regarding the consequences for economic activity, equations (40), (41), (42), (C5) and (C6) in Appendix C imply:

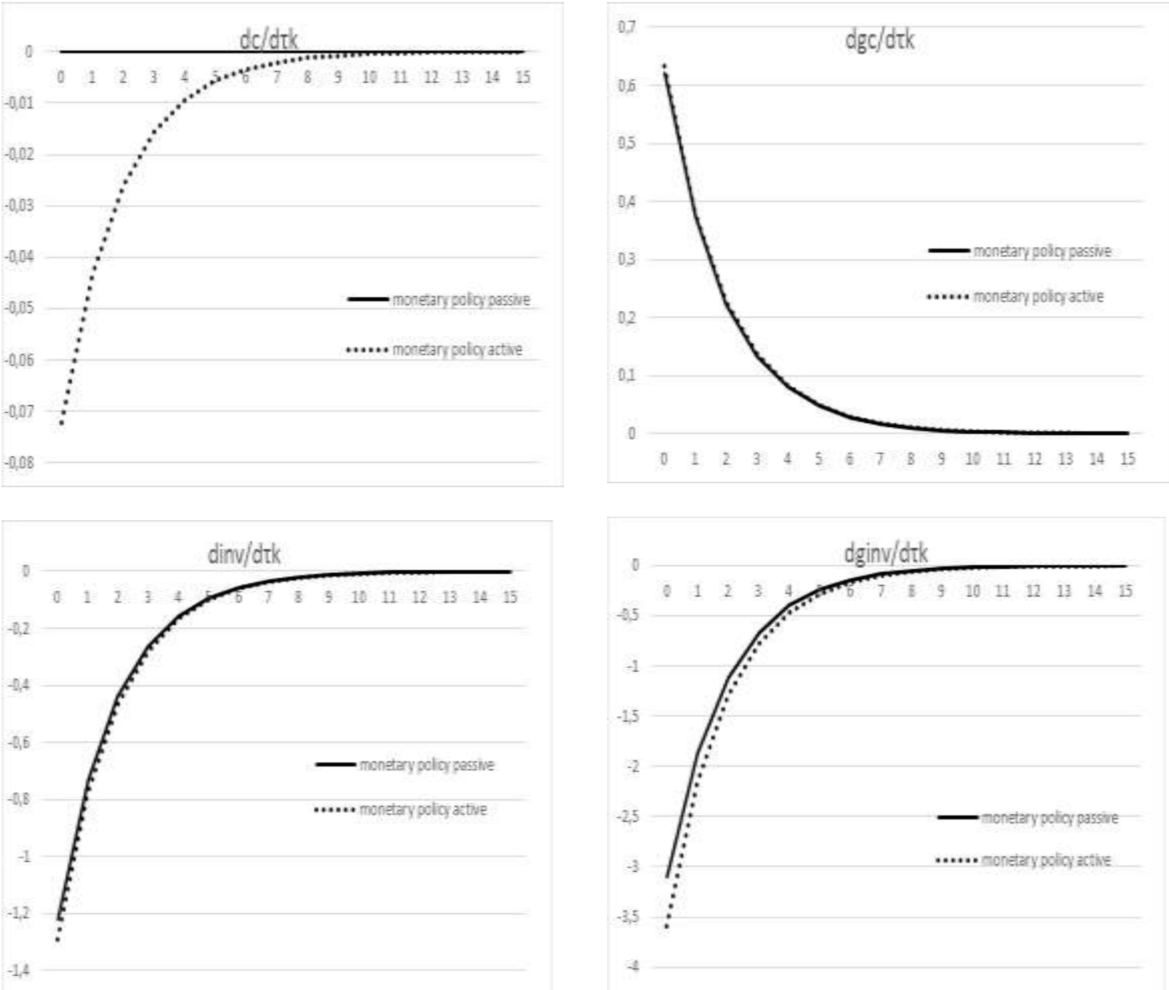
$$\frac{\partial inv_T}{\partial \widehat{\tau_{k,T}}} = -\frac{(1-\delta)k_2\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (61)$$

$$\frac{\partial c_T}{\partial \widehat{\tau_{k,T}}} = \frac{\partial g_T}{\partial \widehat{\tau_{k,T}}} = -\frac{\theta\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (62)$$

$$\begin{aligned}\frac{\partial g_{inv,T}}{\partial \widehat{\tau_{k,T}}} &= \frac{-\left[(1+\varphi\theta)G_{c,T}-\nu(1-\theta)G_{c,T}-\theta(1+\varphi)z_2G_T\right](1-\beta)\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]}\\ &-\frac{\left[(1-\nu)(1+\varphi\theta+\nu\theta)G_{c,T}-\theta(1+\varphi)z_2G_T\right]\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\delta\beta\left(1-\tau_{k,T}\right)}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]}\\ &-\frac{k_2\{[(1+\nu\varphi)-\nu(1-\nu)(1-\delta)]\delta\beta\left(1-\tau_{k,T}\right)+[1-\nu+\nu\delta(1+\varphi)](1-\beta)\}G_{c,T}}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (63)\end{aligned}$$

$$\begin{aligned}\frac{\partial g_{c,T}}{\partial \widehat{\tau_{k,T}}} &= \frac{\left[(1+\theta\varphi)G_{inv,T}-\nu(1-\theta)G_{inv,T}-\theta(1+\varphi)z_1G_T\right]\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)(1-\beta)}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]}\\ &+\frac{\left[(1-\nu)(1+\theta\varphi+\theta\nu)G_{inv,T}-\theta(1+\varphi)z_1G_T\right]\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\delta\beta\left(1-\tau_{k,T}\right)}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]}\widehat{\tau_{k,T}}\\ &+\frac{k_2\{[(1+\nu\varphi)-\nu(1-\nu)(1-\delta)]\delta\beta\left(1-\tau_{k,T}\right)+[1-\nu+\nu\delta(1+\varphi)](1-\beta)\}G_{inv,T}}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \quad (64)\end{aligned}$$

So, regarding economic activity, equations (61) and (63) show that obviously, a decrease in the capital taxation rate and in the capital cost would mainly favor investment. Indeed, with our basic calibration, if the capital taxation rate decreases by (-1%), public investment would increase by 3.1% in the first period (3.6% if monetary policy is active), whereas private investment would increase by 1.2%. On the contrary, equations (62) and (64) show that private consumption would hardly increase by 0.07% and only if monetary policy is active, while public consumption expenditure would be reduced by (-0.62%).



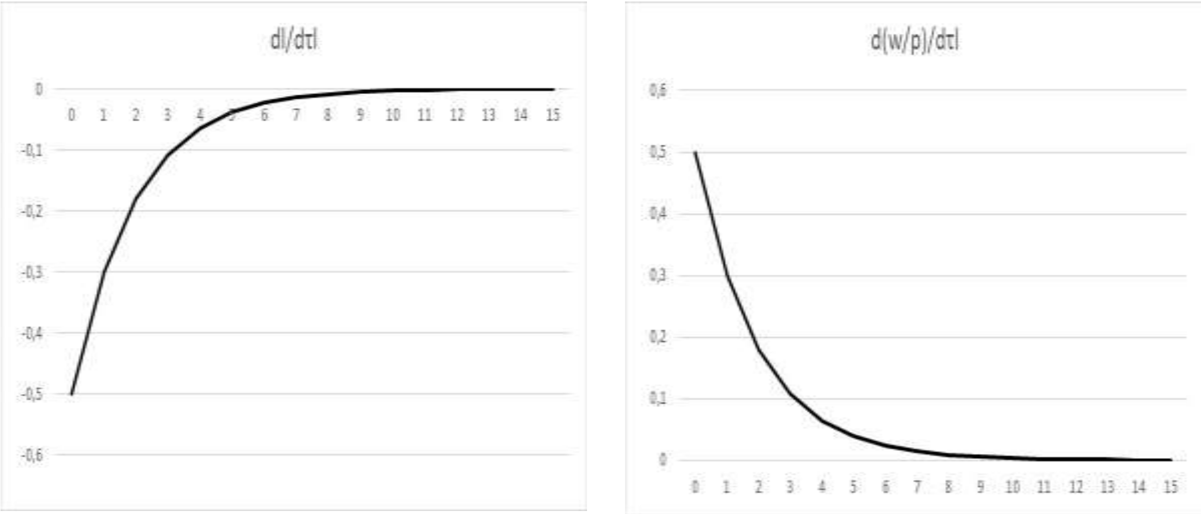
Figures 6: Private and public consumption and investment after a 1% increase in the capital taxation rate (persistence: 0.6)

Therefore, a decrease by (-1%) in the capital taxation rate would favor private and public investment. However, the effect is negative on public consumption expenditure; global economic activity would then only increase by between 0.22% (if monetary policy is passive) and 0.29% (if monetary policy is active) in the first period. Therefore, a decrease in the capital taxation rate would be less efficient than a decrease in the consumption taxation rate in order to sustain economic activity.

4.3 Variation in the labor taxation rate

According to equation (40), the monetary authority doesn't react to a variation in the labor taxation rate, even if monetary policy is active. So, according to equations (C7) and (C8), the main consequence of a decrease in the labor taxation rate is to increase employment, labor supplied by households (less leisure), and to decrease the real wage in order to compensate for this shift in relative preferences. Indeed, we obtain:

$$\frac{\partial l_T}{\partial \widehat{\tau_{l,T}}} = -\frac{1}{(1+\varphi)} \qquad (65) \qquad\qquad \frac{\partial (w_T - p_T)}{\partial \widehat{\tau_{l,T}}} = \frac{1}{(1+\varphi)} \qquad (66)$$



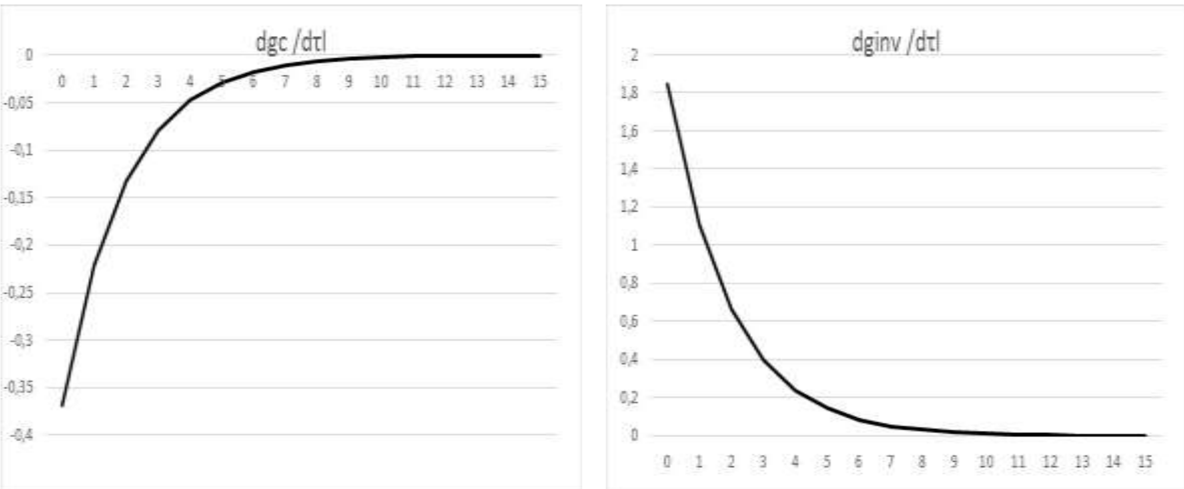
Figures 7: Labor and real wage after a 1% increase in the labor taxation rate (persistence: 0.6)

However, the global income of households, global economic activity, private consumption and private investment remain quite unchanged. According to equations (40), (41) and (42), regarding economic activity, the main implication of a variation in the labor taxation rate would be to shift the composition of public expenditure. Indeed, we obtain:

$$\frac{\partial g_{inv,T}}{\partial \widehat{\tau_{l,T}}} = \frac{(1-\nu)G_{c,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)} \quad (67)$$

$$\frac{\partial g_{c,T}}{\partial \widehat{\tau_{l,T}}} = -\frac{(1-\nu)G_{inv,T}}{\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)(1+\varphi)} \quad (68)$$

So, with our basic calibration, if the labor taxation rate decreases by (-1%), public investment would decrease by (-1.84%) in the first period. Nevertheless, this would be compensated by the stronger labor supply in the production function, and the global production level could remain unchanged. On the contrary, public consumption expenditure would increase by 0.37% in the first period.



Figures 8: Public investment and consumption expenditure after a 1% increase in the labor taxation rate (persistence: 0.6)

Therefore, a decrease in the labor taxation rate would not be able to increase economic activity, in the framework of our model. Private economic activity would remain unchanged, while it would favor public economic consumption to the detriment of public investment.

5 When the budgetary policy is active

This section will now study the consequences of the budgetary constraint. Indeed, beyond the optimal level of budgetary expenditure, we shall consider that the budgetary authority adjusts variations in public expenditure to variations in taxation rates, but taking into account the public debt level. Indeed, it is legitimate to consider that the budgetary authority must avoid an out-bidding of the public indebtedness level, and tries to keep its budget in balance. With such a goal, economic activity and inflation, private consumption and investment, the cost and the level of the capital stock remain unchanged, in the framework of our model. Therefore, the main consequence is to influence labor demand and supply and the real wage, as well as the respective levels of public investment and consumption.

5.1 Variation in the consumption taxation rate

When the government tries to keep its budget in balance, the decrease in the consumption taxation rate necessitates an increase in the labor taxation rate in order to stabilize the level of fiscal resources. Indeed, equation (44) implies:

$$\frac{\partial \widehat{\tau_{l,t}}}{\partial \widehat{\tau_{c,t}}} = -\frac{1}{(1-\nu)}\left(\frac{C_t}{Y_t}\right) \qquad (69)$$

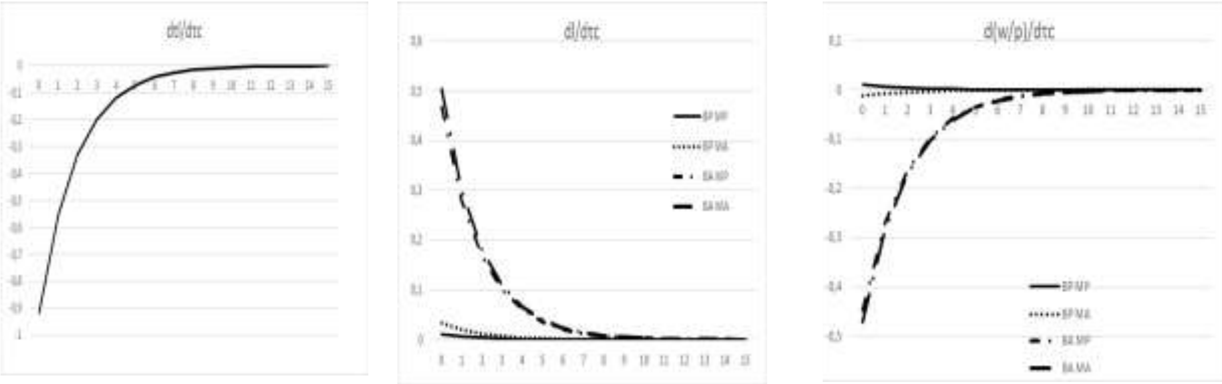
For example, if the consumption taxation rate decreases by (-1%), the labor taxation rate should increase by 0.91% in the first period according to our basic calibration.

In this framework, equations (40), (69), (C7) and (C8) in Appendix C imply:

$$\frac{\partial (w_T - p_T)}{\partial \widehat{\tau_{c,T}}} = -\frac{1}{(1+\varphi)(1-\nu)}\left(\frac{C_t}{Y_t}\right) + \frac{\theta\left(\delta\varphi k_2 - \lambda_{x,CB} - k_1 k_2 \lambda_{\pi,CB}\right)}{(1+\varphi)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \quad (70)$$

$$\frac{\partial l_T}{\partial \widehat{\tau_{c,T}}} = \frac{1}{(1+\varphi)(1-\nu)}\left(\frac{C_t}{Y_t}\right) + \frac{\theta\left(\delta k_2 + \lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)}{(1+\varphi)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \quad (71)$$

Therefore, the higher labor taxation rate implies that the real wage is higher. With our basic calibration, if the consumption taxation rate decreases by (-1%), and if the budgetary policy is active and takes into account the public indebtedness level, the real wage can increase by 0.47% in the first period, instead of remaining quite unchanged. In these conditions, labor demand decreases by (-0.5%) in the first period, instead of hardly decreasing: there is a substitution of capital to labor (the latter has become more expansive).

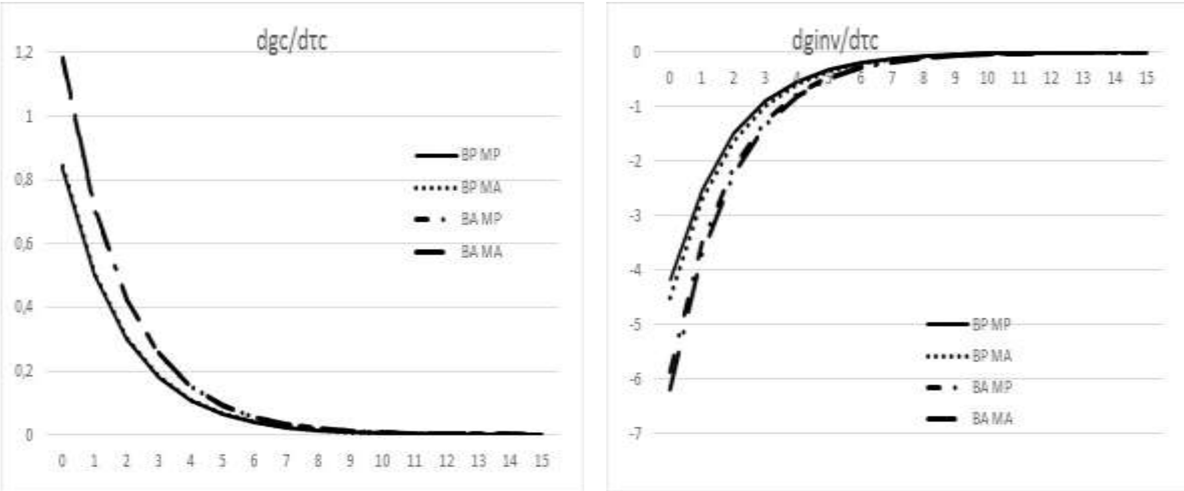


Figures 9: Labor taxation rate, labor supply and demand and real wage after a 1% increase in the consumption taxation rate (persistence: 0.6)

Besides, economic variables still vary in proportion to the consumption taxation rate. Indeed, after a (-1%) decrease in the consumption taxation rate, private investment still increase by around 0.8%, and private consumption by around 1.1% in the first period. Therefore, regarding economic activity, the main implication of a variation in the consumption taxation rate would be to change the composition of public expenditure to the benefit of public investment. Indeed, according to equations (40), (45) and (46), we obtain:

$$\frac{\partial g_{inv,T}}{\partial \widehat{\tau_{c,T}}} = -\frac{G_{c,T}}{(1+\varphi)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)}\left(\frac{C_t}{Y_t}\right) - \frac{\theta\left\{k_2(1+\varphi-\delta\varphi+\delta\varphi\nu)G_{c,T} + \left(\lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)\left[(1-\nu)G_{c,T} - \theta(1+\varphi)z_2 G_T\right]\right\}}{(1+\varphi)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \quad (72)$$

$$\frac{\partial g_{c,T}}{\partial \widehat{\tau_{c,T}}} = \frac{G_{inv,T}}{(1+\varphi)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)}\left(\frac{C_T}{Y_T}\right) + \frac{\theta\left\{k_2(1+\varphi-\delta\varphi+\delta\varphi\nu)G_{inv,T} + \left(\lambda_{x,CB} + k_1 k_2 \lambda_{\pi,CB}\right)\left[(1-\nu)G_{inv,T} - \theta(1+\varphi)z_1 G_T\right]\right\}}{(1+\varphi)\left(z_1 G_{c,T} - z_2 G_{inv,T}\right)\left[(1-\delta)k_2 - \left(\lambda_{x,CB} + \lambda_{\pi,CB} k_1 k_2\right)\theta\right]} \left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\} \quad (73)$$



Figures 10: Public investment and public consumption expenditure after a 1% increase in the consumption taxation rate (persistence: 0.6)

So, according to our basic calibration, if the consumption taxation rate decreases by (-1%), public investment can increase until 6.18% in the first period when monetary policy is active and when the government takes into account the necessity to stabilize the public indebtedness level (instead of 4.5% when the budgetary policy is passive). On the contrary, public consumption expenditure can decrease by (-1.18%) in the first period (instead -0.84%). Indeed, as the budgetary constraint implies a shift in economic preferences in favor of capital and to the detriment of labor, public investment is relatively favored by the government [The first terms in equations (72) and (73) are the only differences with equations (54) and (55)].

5.2 Variation in the capital taxation rate

When the government tries to keep its budget in balance, the decrease in the capital taxation rate necessitates an increase in the labor taxation rate in order to stabilize the level of fiscal resources. Indeed, equation (44) implies:

$$\frac{\partial \widehat{\tau_{l,t}}}{\partial \widehat{\tau_{k,t}}} = -\frac{(1-\beta)\nu}{(1-\nu)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)} \qquad (74)$$

For example, if the capital taxation rate decreases by (-1%), the labor taxation rate should increase by 0.17% in the first period according to our basic calibration.

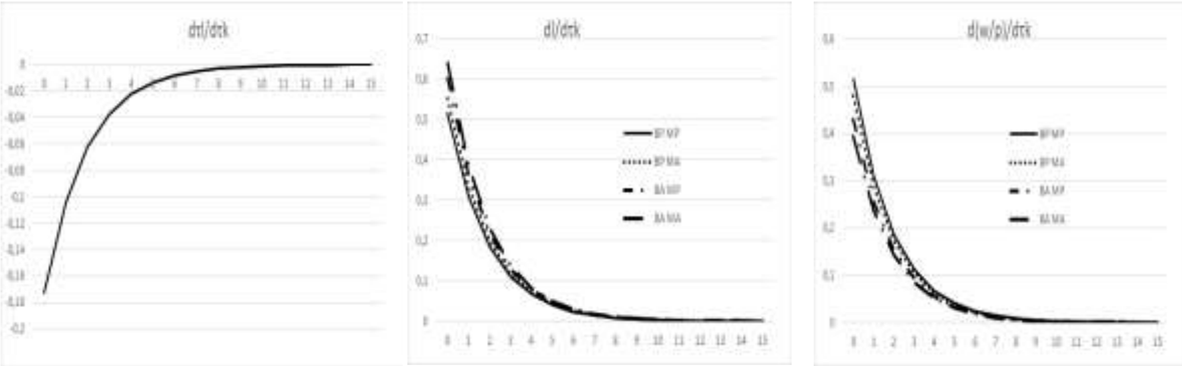
In this framework, equations (40), (74), (C7) and (C8) in Appendix C imply:

$$\frac{\partial l_T}{\partial \widehat{\tau_{k,T}}} = \frac{1}{(1+\varphi)}\left\{\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)(1-\nu+\delta\nu)\right]}{(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}+\frac{(1-\beta)\nu}{(1-\nu)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)}\right\}$$
$$+\frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)(1-\delta+\delta\theta)\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{(1+\varphi)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \qquad (75)$$

$$\frac{\partial (w_T-p_T)}{\partial \widehat{\tau_{k,T}}} = \frac{1}{(1+\varphi)}\left\{\frac{\varphi\left[(1-\beta)+\delta\beta(1-\nu+\nu\delta)\left(1-\tau_{k,T}\right)\right]}{(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}-\frac{(1-\beta)\nu}{(1-\nu)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)}\right\}$$
$$-\frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)(1-\delta-\delta\varphi\theta)\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{(1+\varphi)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} \qquad (76)$$

Therefore, as the fiscal weight decreases on capital but increases on labor, the substitution of capital to labor (which still becomes more expansive) is accentuated when the government takes into account its budgetary constraint. With our basic calibration, if the capital taxation rate decreases by (-

1%), labor demand decreases by (-0.64%) in the first period, (instead of decreasing by -0.52%), if the budgetary policy is active and takes into account the public indebtedness level. The reduction in the real wage should then be weaker, in order to sustain the relative labor demand. The real wage should decrease by around (-0.39%) in the first period (instead of -0.52%).

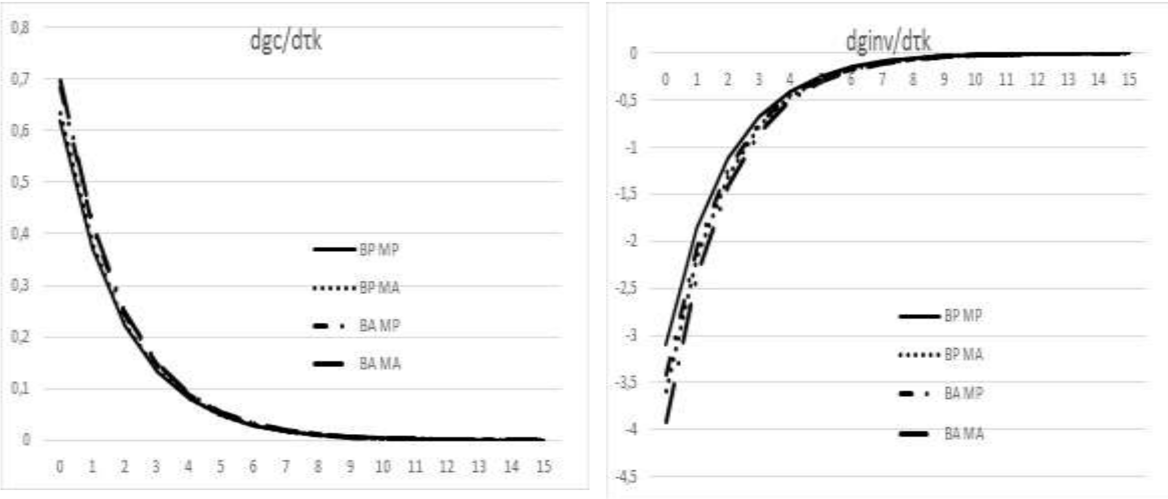


Figures 11: Labor taxation rate, labor supply and demand and real wage after a 1% increase in the capital taxation rate (persistence: 0.6).

Besides, regarding economic activity, equations (61) and (63) show that obviously, a decrease in the capital taxation rate and in the capital cost would mainly favor investment. Indeed, with our basic calibration, if the capital taxation rate decreases by (-1%), private investment would increase by 1.2%, whereas private consumption would hardly vary. Therefore, regarding economic activity, the main implication of a variation in the capital taxation rate would be to change the composition of public expenditure to the benefit of public investment. More precisely, according to equations (40), (45) and (46), we obtain:

$$\frac{\partial g_{inv,T}}{\partial \widehat{\tau_{k,T}}} = -\frac{\left[(1-\nu)(1-\delta-\delta\varphi\theta)G_{c,T}+\theta(1+\varphi)G_{c,T}-\theta(1-\delta)(1+\varphi)z_2 G_T\right]}{(1-\delta)\left(z_1 G_{c,T}-z_2 G_{inv,T}\right)(1+\varphi)} \cdot \frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} - \frac{\left\{\delta\beta\left(1-\tau_{k,t}\right)\left[1+\nu\varphi-\nu(1-\nu)(1-\delta)\right]+(1+\nu\varphi)(1+\nu\delta)(1-\beta)\right\}\delta\beta\left(1-\tau_{k,T}\right)G_{c,T}}{\left(z_1 G_{c,T}-z_2 G_{inv,T}\right)(1+\varphi)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)(1-\delta)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]} - \frac{(1-\beta)(1+\nu\delta\varphi)G_{c,T}}{\left(z_1 G_{c,T}-z_2 G_{inv,T}\right)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)(1+\varphi)(1-\delta)} \qquad (77)$$

$$\frac{\partial g_{c,T}}{\partial \widehat{\tau_{k,T}}} = \frac{\left[(1-\nu)(1-\delta-\delta\varphi\theta)G_{inv,T}+\theta(1+\varphi)G_{inv,T}-\theta(1-\delta)(1+\varphi)z_1 G_T\right]}{\left(z_1 G_{c,T}-z_2 G_{inv,T}\right)(1+\varphi)(1-\delta)} \cdot \frac{\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]\left[(1-\delta)k_2-\left(\lambda_{x,CB}+\lambda_{\pi,CB}k_1k_2\right)\theta\right]} + \frac{\left\{\delta\beta\left(1-\tau_{k,t}\right)\left[1+\nu\varphi-\nu(1-\nu)(1-\delta)\right]+(1+\nu\varphi)(1+\nu\delta)(1-\beta)\right\}\delta\beta\left(1-\tau_{k,T}\right)G_{inv,T}}{(1+\varphi)(1-\delta)\left(z_1 G_{c,T}-z_2 G_{inv,T}\right)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]} + \frac{(1-\beta)(1+\nu\delta\varphi)G_{inv,T}}{\left(z_1 G_{c,T}-z_2 G_{inv,T}\right)(1+\varphi)(1-\delta)\left(1-\beta+\delta\beta-\delta\beta\tau_{k,t}\right)} \qquad (78)$$



Figures 12: Public investment and public consumption expenditure after a 1% increase in the capital taxation rate (persistence: 0.6)

So, according to our basic calibration, if the capital taxation rate decreases by (-1%), public investment can increase until 3.9% in the first period when the government takes into account the necessity to stabilize the public indebtedness level and when monetary and budgetary policies are both active (instead of 3.6% when the budgetary policy is passive). On the contrary, public consumption expenditure can decrease by (-0.7%) in the first period (instead of -0.63%). Indeed, as the budgetary constraint implies a shift in economic preferences in favor of capital and to the detriment of labor, public investment is still more favored by the government.

6 Conclusion

We have used a simple DSGE model in order to evaluate the efficiency of various fiscal policies in order to sustain economic activity and growth. In this framework, a decrease in the consumption taxation rate appears as the most efficient fiscal policy. Indeed, as goods are less expansive, it implies an increase which is more than proportional in private consumption, and also an increase in private investment. Besides, it strongly favors public investment in the composition of public expenditure, in order to increase the productivity of private factors, economic production, and to satisfy the higher global demand, whereas public consumption expenditure decreases. These basic results are still valid if the government takes into account the budgetary constraint and the necessity to balance its budget and to avoid an out-bidding of its indebtedness level.

In comparison, a decrease in the capital taxation rate would decrease the capital cost, and it would favor private and public investment. However, the effect is minor on private consumption and even negative on public consumption expenditure. If the government takes into account the budgetary constraint, the substitution of capital to labor is still accentuated, and public investment would still be more favored in the composition of public expenditure. However, the increase in global economic activity is then more moderate than in the case of a decrease in the consumption taxation rate. Finally, a decrease in the labor taxation rate would not be able to increase economic activity, in the framework of our model, despite the decrease in the real wage. Private economic activity would then remain unchanged, while it would favor public consumption to the detriment of the most productive public investment expenditure.

Therefore, this paper gives interesting indications regarding the efficiency of reductions in various taxation rates in order to sustain economic activity. Nevertheless, future researches could include the following directions. We would like to study the consequences of the introduction of rule of thumb consumers who cannot optimize their consumption level but who simply consume their disposable income, as well as transfers from the government to these agents. We would like to study the consequences of nominal wages rigidities on the labor market, due to the presence of trade-unions for example, avoiding large variations in nominal wages. Finally, we would like to study the consequences of the introduction of an open-economy framework, where households consume both domestically produced and foreign goods, allowing a differential between producer and consumer prices.

References

Alesina A. and S. Ardagna (2010) Large Changes in Fiscal Policy: Taxes versus Spending. In J. R. Brown (ed.), *Tax Policy and the Economy*, vol.24, 35-68.
Ardagna S. (2004) Fiscal Stabilizations: When do they Work and Why?. *European Economic Review*, vol.48, n°5, 1047-1074.
Bhattarai K. and D. Trzeciakiewicz (2017) Macroeconomic Impacts of Fiscal Policy Shocks in the UK: A DSGE Analysis. *Economic Modelling*, vol.61, February, 331-338.
Baxter M. and R. King (1993) Fiscal Policy in General Equilibrium, *American Economic Review*, vol.83, n°3, June, 315-334.
Blanchard O. and R. Perotti (2002) An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. *The Quarterly Journal of Economics*, vol.117, n°4, 1329-68.
Bouakez H. and N. Rebei (2007) Why does Private Consumption Rise after a Government Spending Shock?. *Canadian Journal of Economics*, vol.40, n°3, 954–979.
Burnside C., M. Eichenbaum and J. Fisher (2004) Fiscal Shocks and their Consequences. *Journal of Economic Theory*, vol.115, n°1, March, 89-117.
Carvalho V. M. and M. M. F. Martins (2011) Macroeconomic Effects of Fiscal Consolidations in a DSGE Model for the Euro Area: Does Composition Matter?. FEP Working Papers, n°421, Universidade do Porto, Facultade de Economia do Porto.
Coenen G. and R. Straub (2005) Does Government Spending Crowd in Private Consumption? Theory and Empirical Evidence for the Euro Area. *International Finance*, vol.8, n°3, 435-470.
Coenen G., M. Mohr and R. Straub (2008) Fiscal Consolidation in the Euro Area: Long-Run Benefits and Short-Run Costs. *Economic Modelling*, vol.25, n°5, 912-932.
Davig T. and E. M. Leeper (2011) Monetary-Fiscal Policy Interactions and Fiscal Stimulus. *European Economic Review*, vol.55, n°2, February, 211–227.
De Haan J. and W. Romp (2005) Public Capital and Economic Growth: A Critical Survey. *European Investment Bank Papers*, vol.10, n°1, 40-71.
Drautzberg T. and H. Uhlig (2011) Fiscal Stimulus and Distortionary Taxation. The Milton Friedman Institute for Research in Economics, Discussion Paper, n°2011-005.
Edelberg W., M. Eichenbaum and J. Fisher (1999) Understanding the Effects of a Shock to Government Purchases. *Review of Economics Dynamics*, vol.2, n°1, January, 166-206.
Fatas A. and I. Mihov (2001) The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence. CEPR Discussion Papers, n°2760.
Finn M. G. (1998) Cyclical Effects of Government's Employment and Goods Purchases. *International Economic Review*, vol.39, n°3, August, 635-657.
Forni L., L. Monteforte and L. Sessa (2009) The General Equilibrium Effects of Fiscal Policy: Estimates for the Euro Area. *Journal of Public Economics*, vol.93, n°3-4, 559-585.
Furlanetto F. (2007) Fiscal Shocks and the Consumption Response when Wages are Sticky. Cahiers de Recherches Economiques du Département d'Econométrie et d'Economie Politique (DEEP) 07.11, Université de Lausanne, Faculté des HEC.
Galí J. (2008) *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton: Princeton University Press.
Galí J., J. D. López-Salido and J. Vallés (2007) Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association*, vol.5, n°1, 227-270
Leeper E. M., T. B. Walker and S.-C. S. Yang (2010) Government Investment and Fiscal Stimulus. *Journal of Monetary Economics*, vol.57, n°8, 1000–1012.
Leeper E. M., N. Traum and T. B. Walker (2011) Clearing up the Fiscal Multiplier Morass. NBER Working paper, n°17444, September.

Ludvigson S. (1996) The Macroeconomic Effects of Government Debt in a Stochastic Growth Model. *Journal of Monetary Economics*, vol.38, n°1, August, 25-45.

Mertens K. and M. O. Ravn (2011) Understanding the Aggregate Effects of Anticipated and Unanticipated Tax Policy Shocks. *Review of Economic Dynamics,* vol.14, n°1: 27-54.

Pappa E. (2004) New Keynesian or RBC Transmission? The Effects of Fiscal Policy in Labor Markets. Working Papers 293, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.

Pappa E. (2009) The Effects of Fiscal Shocks on Employment and the Real Wage. *International Economic Review*, vol.50, n°1, February, 217-244.

Perotti R. (2004) Public Investment: Another (Different) Look. Working Papers 2977, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.

Sims E. and J. Wolff (2013) The Output and Welfare Effects of Government Spending Shocks over the Business Cycle. NBER Working Papers, n°19749, December.

Smets F. and R. Wouters (2003) An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, vol.1, n°5, 1123-1175.

Straub R. and I. Tchakarov (2007) Assessing the Impact of Change in the Composition of Public Spending: A DSGE Approach. ECB Working Paper Series, n°795, European Central Bank, August.

Woodford M. (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy,* Princeton: Princeton University Press.

Zubairy S. (2014) On Fiscal Multipliers: Estimates from a Medium Scale DSGE Model. *International Economic Review*, vol.55, n°1, February, 169-195.

Appendix A: Optimal output-gap, inflation and capital stock

Equations (6), (34) and (B7) imply: $k_{T+1} = k_T - \delta k_2 x_T \qquad (A1)$

Equations (32), (A1) and (B7) imply:

$$\overline{r_T} = -\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}}$$
$$+\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T+1}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T+1}\right)\right]}E_T\left(\frac{G_{T+1}}{Y_{T+1}}\right)\right\}E_T\left(\widehat{\tau_{c,T+1}}\right)$$
$$+\frac{1}{\theta}(1 + k_2 - \delta k_2)x_T - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\theta\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}}$$
$$-\frac{1}{\theta}(1 + k_2)E_T(x_{T+1}) + \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T+1}\right)\right]}{\theta\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T+1}\right)\right]}E_T\left(\widehat{\tau_{k,T+1}}\right) \qquad (A2)$$

Therefore, equations (32) and (A2) imply:

$$x_T = \frac{1}{(1-\delta)}E_T(x_{T+1}) - \frac{\theta}{(1-\delta)k_2}E_T(\pi_{T+1}) + \frac{1}{(1-\delta)}\lambda_T \qquad (A3)$$

$$\lambda_T = \frac{\theta}{k_2}i_T + \frac{\theta}{k_2}\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}}$$
$$-\frac{\theta}{k_2}\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T+1}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T+1}\right)\right]}E_T\left(\frac{G_{T+1}}{Y_{T+1}}\right)\right\}E_T\left(\widehat{\tau_{c,T+1}}\right)$$
$$+\frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{k_2\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}}$$
$$-\frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T+1}\right)\right]}{k_2\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T+1}\right)\right]}E_T\left(\widehat{\tau_{k,T+1}}\right)$$

Afterwards, equations (38) and (A3) imply:

$$\pi_T = [\beta - \frac{\theta k_1}{(1-\delta)}]E_t(\pi_{T+1}) + \frac{k_1 k_2}{(1-\delta)}E_T(x_{T+1}) + \frac{k_1 k_2}{(1-\delta)}\lambda_T \qquad (A4)$$

Equations (A1) and (A3) imply:

$$k_T = E_T(k_{T+1}) + \frac{\delta k_2}{(1-\delta)}E_T(x_{T+1}) - \frac{\delta\theta}{(1-\delta)}E_T(\pi_{T+1}) + \frac{\delta k_2}{(1-\delta)}\lambda_T \qquad (A5)$$

So, this implies to solve the following system:

$$\begin{pmatrix} x_T \\ \pi_T \\ k_T \end{pmatrix} = A\begin{pmatrix} E_T(x_{T+1}) \\ E_T(\pi_{T+1}) \\ E_T(k_{T+1}) \end{pmatrix} + \frac{1}{(1-\delta)}\begin{pmatrix} \lambda_T \\ k_1 k_2 \lambda_T \\ \delta k_2 \lambda_T \end{pmatrix}$$

With: $\lim_{n\to\infty} x_n = \lim_{n\to\infty} \pi_n = \lim_{n\to\infty} k_n = 0$, this implies:

$$\begin{pmatrix} x_T \\ \pi_T \\ k_T \end{pmatrix} = \frac{1}{(1-\delta)}\sum_{n=T}^{\infty} A^{n-T}\begin{pmatrix} \lambda_n \\ k_1 k_2 \lambda_n \\ \delta k_2 \lambda_n \end{pmatrix} \qquad (A6)$$

Then, we have to find the solution of the following matrix:

$$A^n = \frac{1}{(1-\delta)^n}\begin{pmatrix} 1 & -\frac{\theta}{k_2} & 0 \\ k_1 k_2 & (\beta - \beta\delta - \theta k_1) & 0 \\ \delta k_2 & -\delta\theta & (1-\delta) \end{pmatrix}^n = \begin{pmatrix} a_n & b_n & c_n \\ d_n & e_n & f_n \\ g_n & h_n & j_n \end{pmatrix}$$

$$with;\quad u_n = \left[\beta + \frac{(2 - \delta - \theta k_1)}{(1-\delta)}\right]u_{n-1} - \left[\beta + \frac{(1 + \beta - \theta k_1)}{(1-\delta)}\right]u_{n-2} + \frac{\beta}{(1-\delta)}u_{n-3}$$

as characteristic equation of this matrix.

Besides, we obtain the following economic variables:

$$x_T = \frac{1}{(1-\delta)} \sum_{n=T}^{\infty} (a_{n-T} + k_1 k_2 b_{n-T} + \delta k_2 c_{n-T}) \lambda_n \qquad (A7)$$

$$\pi_T = \frac{1}{(1-\delta)} \sum_{n=T}^{\infty} (d_{n-T} + k_1 k_2 e_{n-T} + \delta k_2 f_{n-T}) \lambda_n \qquad (A8)$$

$$k_T = \frac{1}{(1-\delta)} \sum_{n=T}^{\infty} (g_{n-T} + k_1 k_2 h_{n-T} + \delta k_2 j_{n-T}) \lambda_n \qquad (A9)$$

- $a_0 = 1 \qquad a_1 = \frac{1}{(1-\delta)} \qquad a_2 = \frac{(1-\theta k_1)}{(1-\delta)^2} \qquad a_3 = \frac{(1-\theta k_1)^2}{(1-\delta)^3} - \frac{\beta\theta k_1}{(1-\delta)^2}$ .....
- $b_0 = 0 \qquad b_1 = -\frac{\theta}{(1-\delta)k_2} \qquad b_2 = -\frac{\theta(1+\beta-\beta\delta-\theta k_1)}{(1-\delta)^2 k_2}$

  $b_3 = -\frac{\theta[(1-\theta k_1)^2 + \beta(1-\delta)(1+\beta-\beta\delta-2\theta k_1)]}{(1-\delta)^3 k_2}$ .....
- $(\forall n) \qquad c_n = f_n = 0 \qquad d_n = -\frac{k_1 k_2^2}{\theta} b_n \qquad j_n = 1$
- $e_0 = 1 \qquad e_1 = \left(\beta - \frac{\theta k_1}{(1-\delta)}\right) \qquad e_2 = \frac{[(\beta-\beta\delta-\theta k_1)^2 - \theta k_1]}{(1-\delta)^2}$

  $e_3 = -\frac{\theta k_1(1-\theta k_1)^2}{(1-\delta)^3} + \frac{\beta[\beta^2(1-\delta)^2 - (2-3\theta k_1+3\beta-3\beta\delta)\theta k_1]}{(1-\delta)^2}$ ...
- $g_0 = 0 \qquad g_1 = \frac{\delta k_2}{(1-\delta)} \qquad g_2 = \frac{\delta k_2(2-\theta k_1-\delta)}{(1-\delta)^2}$

  $g_3 = [(2-\delta-\theta k_1)(1-\theta k_1) + (1-\delta-\beta\theta k_1)(1-\delta)]\frac{\delta k_2}{(1-\delta)^3}$ ...
- $h_0 = 0 \qquad h_1 = -\frac{\delta\theta}{(1-\delta)} \qquad h_2 = -\frac{\delta\theta(2-\delta+\beta-\beta\delta-\theta k_1)}{(1-\delta)^2}$

  $h_3 = -\frac{\delta\theta}{(1-\delta)^3}[\beta(1-\delta)(2+\beta-\beta\delta-2\theta k_1-\delta) + (1-\theta k_1)(2-\theta k_1-\delta) + (1-\delta)^2]$ ...

Appendix B: Definition of all other economic variables

Real capital cost: according to equation (7): $\left(r_T^k - p_T\right) = \widehat{\tau_{k,T}}$

Equations (15), (36) and (37) imply:

$$(1 - z_1 - z_2)y_T^p = a_T - (1 - z_1 - z_2)x_T + \nu k_T + (1 - \nu)l_T + \left(z_1 - z_2\frac{G_{inv,T}}{G_{c,T}}\right)\varepsilon_T^{g,inv} + z_2\frac{G_T}{G_{c,T}}\frac{\delta\nu\beta\left(1 - \tau_{k,T}\right)}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} + z_2\theta\left(\frac{G_T}{G_{c,T}}\right)\left\{1 - \left(\frac{G_T}{Y_T}\right)\frac{\left[(1 - \beta) + \delta\beta\left(1 - \tau_{k,T}\right)\right]}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\right\}\widehat{\tau_{c,T}} \quad (B1)$$

Equation (38) implies:

$$\left(z_1 - z_2\frac{G_{inv,T}}{G_{c,T}}\right)\varepsilon_T^{g,inv} = \frac{k_2(1 + \nu\varphi)}{(1 + \varphi)}y_T^p - a_T + \frac{(1 - \nu)}{(1 + \varphi)}\widehat{\tau_{l,T}} + \frac{\left[(1 - \nu)G_{c,T} - \theta(1 + \varphi)z_2G_T\right]}{(1 + \varphi)G_{c,T}}\left\{1 - \frac{\left[(1 - \beta) + \delta\beta\left(1 - \tau_{k,T}\right)\right]}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} + \nu\widehat{\tau_{k,T}} - \frac{\delta\nu\beta\left(1 - \tau_{k,T}\right)\left[\theta(1 + \varphi)z_2G_T - (1 - \nu)G_{c,T}\right]}{\theta(1 + \varphi)\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]G_{c,T}}\widehat{\tau_{k,T}} \quad (B2)$$

By combining equations (B1) and (B2), and using equation (k_2) in (38), we have:

$$y_T^p = -\frac{\theta(1 + \varphi)(1 - z_1 - z_2)}{\left[\theta(1 + \varphi) - (1 - \nu)(1 + \varphi\theta)\right]}x_T + \frac{\nu\theta(1 + \varphi)}{\left[\theta(1 + \varphi) - (1 - \nu)(1 + \varphi\theta)\right]}k_T + \frac{\theta(1 - \nu)}{\left[\theta(1 + \varphi) - (1 - \nu)(1 + \varphi\theta)\right]}\left[(1 + \varphi)l_T + \widehat{\tau_{l,T}}\right] + \frac{\theta(1 - \nu)}{\left[\theta(1 + \varphi) - (1 - \nu)(1 + \varphi\theta)\right]}\left\{1 - \frac{\left[(1 - \beta) + \delta\beta\left(1 - \tau_{k,T}\right)\right]}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} + \frac{\nu\left[(1 - \beta)\theta(1 + \varphi) + \delta\beta(1 + \theta + \theta\varphi)(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}{\left[\theta(1 + \varphi) - (1 - \nu)(1 + \varphi\theta)\right]\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \quad (B3)$$

Equations (7), (13), (17) and (35) imply:

$$l_T = \frac{1}{(1 + \varphi)}\left(k_T - \widehat{\tau_{l,T}}\right) - \frac{1}{(1 + \varphi)}\left\{1 - \frac{\left[(1 - \beta) + \delta\beta\left(1 - \tau_{k,T}\right)\right]}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} - \frac{1}{\theta(1 + \varphi)}\left(x_T + y_T^p\right) + \frac{\left[\theta(1 - \beta) + \delta\beta(\theta - \theta\nu - \nu)\left(1 - \tau_{k,T}\right)\right]}{\theta(1 + \varphi)\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \quad (B4)$$

Therefore, by combining equations (B3) and (B4), and the expression of (k_2) in equation (38), we have the variation in **labor demand and supply**:

$$l_T = \frac{k_2}{\theta(1 + \varphi)}x_T + \frac{(\theta - 1)}{(1 + \varphi)\theta}k_T - \frac{\left[\delta\beta\left(1 - \tau_{k,T}\right)(1 - \theta + \nu\theta) + (1 - \beta)(1 - \theta)\right]}{\theta(1 + \varphi)\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} - \frac{1}{(1 + \varphi)}\widehat{\tau_{l,T}} - \frac{1}{(1 + \varphi)}\left\{1 - \frac{\left[(1 - \beta) + \delta\beta\left(1 - \tau_{k,T}\right)\right]}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} \quad (B5)$$

Thus, by combining equations (7), (17) and (B5), the variation in the **real wage** is:

$$(w_T - p_T) = -\frac{k_2}{\theta(1 + \varphi)}x_T + \frac{(1 + \varphi\theta)}{(1 + \varphi)\theta}k_T + \frac{1}{(1 + \varphi)}\widehat{\tau_{l,T}} + \frac{1}{(1 + \varphi)}\left\{1 - \frac{\left[(1 - \beta) + \delta\beta\left(1 - \tau_{k,T}\right)\right]}{\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} + \frac{\left[\delta\beta\left(1 - \tau_{k,T}\right)(1 + \theta\varphi - \nu\theta\varphi) + (1 - \beta)(1 + \theta\varphi)\right]}{(1 + \varphi)\theta\left[(1 - \beta) + \delta\beta(1 - \nu)\left(1 - \tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \quad (B6)$$

Equations (B3) and (B5) imply the following **potential production level**:

$$y_T^p = -(1+k_2)x_T + k_T + \widehat{\tau_{k,T}} \qquad (B7)$$

So, equations (33), (34), (35), (36), (37), (B2) and (B7) imply:

$$y_T = -k_2 x_T + k_T + \widehat{\tau_{k,T}} \qquad (B8)$$

$$inv_T = -k_2 x_T + k_T \qquad (B9)$$

$$c_T = -k_2 x_T + k_T + \frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} - \frac{\theta\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\widehat{\tau_{c,T}} \qquad (B10)$$

$$\begin{aligned} g_{inv,T} = & -\frac{k_2\left[(1-\nu)G_{c,T}+\theta(1+\varphi)\left(G_{c,T}-z_2G_T\right)\right]}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\theta}x_T - \frac{G_{c,T}}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)}a_T \\ & + \frac{\left[(1-\nu)(1+\varphi\theta)G_{c,T}-\theta(1+\varphi)z_2G_T\right]}{\theta(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)}k_T + \frac{(1-\nu)G_{c,T}}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)}\widehat{\tau_{l,T}} \\ & + \frac{\left[(1-\nu)G_{c,T}-\theta(1+\varphi)z_2G_T\right]}{(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)}\left\{1-\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} \\ & + \frac{\left[(1-\nu+\varphi\theta+\nu\theta)(1-\beta)+\delta\beta(1+\varphi\theta+\nu\theta)(1-\nu)\left(1-\tau_{k,T}\right)\right]G_{c,T}}{(1+\varphi)\theta\left(z_1G_{c,T}-z_2G_{inv,T}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \\ & - \frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]z_2G_T}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \qquad (B11) \end{aligned}$$

$$\begin{aligned} g_{c,T} = & \frac{k_2\left[(1-\nu)G_{inv,T}+\theta(1+\varphi)(G_{inv,T}-z_1G_T)\right]}{\left(z_1G_{c,T}-z_2G_{inv,T}\right)(1+\varphi)\theta}x_T + \frac{G_{inv,T}}{(z_1G_{c,T}-z_2G_{inv,T})}a_T \\ & - \frac{\left[(1-\nu)(1+\varphi\theta)G_{inv,T}-\theta(1+\varphi)z_1G_T\right]}{\theta(1+\varphi)\left(z_1G_{c,T}-z_2G_{inv,T}\right)}k_T - \frac{(1-\nu)G_{inv,T}}{(1+\varphi)(z_1G_{c,T}-z_2G_{inv,T})}\widehat{\tau_{l,T}} \\ & - \frac{[(1-\nu)G_{inv,T}-\theta(1+\varphi)z_1G_T]}{(1+\varphi)(z_1G_{c,T}-z_2G_{inv,T})}\left\{1-\frac{G_T}{Y_T}\frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\right\}\widehat{\tau_{c,T}} \\ & - \frac{\left[(1-\nu+\varphi\theta+\nu\theta)(1-\beta)+\delta\beta(1+\varphi\theta+\nu\theta)(1-\nu)\left(1-\tau_{k,T}\right)\right]G_{inv,T}}{(1+\varphi)\theta\left(z_1G_{c,T}-z_2G_{inv,T}\right)\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \\ & + \frac{\left[(1-\beta)+\delta\beta\left(1-\tau_{k,T}\right)\right]z_1G_T}{(z_1G_{c,T}-z_2G_{inv,T})\left[(1-\beta)+\delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} \qquad (B12) \end{aligned}$$

Appendix C: Main economic variables according to taxation rates

Equations (A3), (A7), (A8) and (A9) imply:

$$x_T = \frac{\theta}{(1-\delta)k_2} i_T + \frac{\theta}{(1-\delta)} \sum_{n=T+1}^{\infty} (\frac{a_{n-T}}{k_2} + k_1 b_{n-T}) i_n + \frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{k_2(1-\delta)[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]} \widehat{\tau_{k,T}}$$
$$+ \sum_{n=T+1}^{\infty} \frac{[(1-\beta)+\delta\beta(1-\tau_{k,n})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,n})]} [\frac{(a_{n-T}-a_{n-T-1})}{(1-\delta)k_2} + \frac{k_1(b_{n-T}-b_{n-T-1})}{(1-\delta)}] \widehat{\tau_{k,n}}$$
$$+ \frac{\theta}{(1-\delta)k_2} \left\{1 - \frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]} \left(\frac{G_T}{Y_T}\right)\right\} \widehat{\tau_{c,T}}$$
$$+ \frac{\theta}{(1-\delta)} \sum_{n=T+1}^{\infty} \left[\frac{(a_{n-T}-a_{n-T-1})}{k_2} + k_1(b_{n-T}-b_{n-T-1})\right] \left\{1 - \frac{[(1-\beta)+\delta\beta(1-\tau_{k,n})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,n})]} \left(\frac{G_n}{Y_n}\right)\right\} \widehat{\tau_{c,n}} \qquad (C1)$$

$$\pi_T = \frac{\theta k_1}{(1-\delta)} i_T - \frac{k_1}{(1-\delta)} \sum_{n=T+1}^{\infty} (k_2 b_{n-T} - \theta e_{n-T})\, i_n + \frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]k_1}{(1-\delta)[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]} \widehat{\tau_{k,T}}$$
$$- \frac{k_1}{(1-\delta)} \sum_{n=T+1}^{\infty} \frac{[(1-\beta)+\delta\beta(1-\tau_{k,n})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,n})]} [\frac{k_2}{\theta}(b_{n-T}-b_{n-T-1}) - (e_{n-T}-e_{n-T-1})] \widehat{\tau_{k,n}}$$
$$+ \frac{\theta k_1}{(1-\delta)} \left\{1 - \frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]} \left(\frac{G_T}{Y_T}\right)\right\} \widehat{\tau_{c,T}}$$
$$- \frac{1}{(1-\delta)} \sum_{n=T+1}^{\infty} [k_1 k_2 (b_{n-T}-b_{n-T-1}) - \theta k_1 (e_{n-T}-e_{n-T-1}] \left\{1 - \frac{[(1-\beta)+\delta\beta(1-\tau_{k,n})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,n})]} \left(\frac{G_n}{Y_n}\right)\right\} \widehat{\tau_{c,n}} \qquad (C2)$$

$$k_T = \frac{\delta\theta}{(1-\delta)} i_T + \frac{\theta}{(1-\delta)} \sum_{n=T+1}^{\infty} \left(\frac{g_{n-T}}{k_2} + k_1 h_{n-T} + \delta\right) i_n + \frac{\delta[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{(1-\delta)[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]} \widehat{\tau_{k,T}}$$
$$+ \sum_{n=T+1}^{\infty} \frac{[(1-\beta)+\delta\beta(1-\tau_{k,n})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,n})]} [\frac{(g_{n-T}-g_{n-T-1})}{k_2(1-\delta)} + \frac{k_1(h_{n-T}-h_{n-T-1})}{(1-\delta)}] \widehat{\tau_{k,n}}$$
$$+ \frac{\delta\theta}{(1-\delta)} \left\{1 - \frac{[(1-\beta)+\delta\beta(1-\tau_{k,T})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,T})]} \left(\frac{G_T}{Y_T}\right)\right\} \widehat{\tau_{c,T}}$$
$$+ \frac{\theta}{(1-\delta)} \sum_{n=T+1}^{\infty} \left[\frac{1}{k_2}(g_{n-T}-g_{n-T-1}) + k_1(h_{n-T}-h_{n-T-1})\right] \left\{1 - \frac{[(1-\beta)+\delta\beta(1-\tau_{k,n})]}{[(1-\beta)+\delta\beta(1-\nu)(1-\tau_{k,n})]} \left(\frac{G_n}{Y_n}\right)\right\} \widehat{\tau_{c,n}} \qquad (C3)$$

Therefore, equations (B8), (B9), (B10), (C1) and (C3) imply:

$$y_T = -\theta i_T - \theta\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} - \frac{\delta\beta\nu\left(1-\tau_{k,T}\right)}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right) \quad (C4)$$

$$inv_T = -\theta i_T - \theta\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right) \quad (C5)$$

$$c_T = -\theta i_T - \theta\widehat{\tau_{c,T}} + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right) \quad (C6)$$

Besides, equations (B5), (B6), (C1) and (C3) imply:

$$l_T = -\frac{1}{(1+\varphi)}\widehat{\tau_{l,T}} + \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)(1-\nu+\delta\nu)\right]}{(1+\varphi)(1-\delta)\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} + \frac{\theta\delta}{(1+\varphi)(1-\delta)}\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} + \frac{(1-\delta+\delta\theta)}{(1+\varphi)(1-\delta)}i_T + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right) \quad (C7)$$

$$(w_T - p_T) = \frac{1}{(1+\varphi)}\widehat{\tau_{l,T}} + \frac{\varphi\left[(1-\beta) + \delta\beta(1-\nu+\nu\delta)\left(1-\tau_{k,T}\right)\right]}{(1+\varphi)(1-\delta)\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\widehat{\tau_{k,T}} + \frac{\delta\varphi\theta}{(1+\varphi)(1-\delta)}\left\{1 - \frac{\left[(1-\beta) + \delta\beta\left(1-\tau_{k,T}\right)\right]}{\left[(1-\beta) + \delta\beta(1-\nu)\left(1-\tau_{k,T}\right)\right]}\left(\frac{G_T}{Y_T}\right)\right\}\widehat{\tau_{c,T}} - \frac{(1-\delta-\delta\varphi\theta)}{(1+\varphi)(1-\delta)}i_T + f\left(\sum_{n=T+1}^{\infty} i_n, \sum_{n=T+1}^{\infty}\widehat{\tau_{c,n}}, \sum_{n=T+1}^{\infty}\widehat{\tau_{k,n}}\right) \quad (C8)$$