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GARCH model and fat tails of the Chinese stock market returns - New evidences

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Abstract

The Chinese stock market is unique in which it is moved more by individual retail investors than institutional investors. Therefore, for economic and political stability it is more important to efficiently manage the risk of the Chinese stock market. We investigate its volatility dynamics through the GARCH model with three types of heavy-tailed distributions, the Student's *t*, the NIG and the NRIG distributions. Our results show that estimated parameters for all the three types of distributions are statistical significant and the NIG distribution has the best empirical performance in fitting the Chinese stock market index returns.

JEL Classification numbers: C22; C52; G17
Key words: generalized hyperbolic distribution, GARCH model, SHA

1 Introduction

The tide of reform and opening up of China's economy since the end of the 1970s, promoted the emergence of China's capital market. After more than 20 years of practice, as joint efforts of the government and the market, China's capital market is making every progress from zero, expanding the bearing capacity, optimizing the structure and function, and constantly improving the system construction. With the growth of market participants, China's capital market has been developing in legal system, trading rules, regulatory systems, which are approaching international standards. Today, China has established a stock market with the 3rd largest market capitalization globally, a bond market with the fifth largest balance globally, and a futures market with trading volume among the highest in the world. China's capital market has become an important platform

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to optimize the allocation of resources, to promote the development pattern, and to promote the sustainable development of China's economy.

The Shanghai Stock Exchange (SSE) is one of the two main stock exchanges operating independently in the People's Republic of China, the other being the Shenzhen Stock Exchange. Shanghai Stock Exchange is the world's 5th largest stock market by market capitalization at US$3.5 trillion as of February 2016, and 2nd largest in East Asia and Asia. The SSE 50 Index (SHA) is the stock indices of the SSE, representing the top 50 by "float-adjusted" capitalization. The SSE 50 is one of the most popular indices for the Chinese stock markets. As pointed out by Cont (2001), two of the eleven stylized facts for asset returns are: volatility clustering and conditional heavy tails. The volatility clustering says that different measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time; and the conditional heavy tails say that even after correcting returns for volatility clustering (e.g. via GARCH-type models), the residual time series still exhibit heavy tails. However, the tails are less heavy than in the unconditional distribution of returns. In this paper, we reconsider the two stylized facts, volatility clustering and conditional heavy tails, but focus on the stock market returns on the Shanghai Stock Exchange.

Our framework is the same as the framework in Guo (2017). Guo compared empirical performance of the Student's *t*, Skewed *t*, normal inverse Gaussian (NIG), and normal reciprocal inverse Gaussian (NRIG) distributions within the genearalized autoregressive conditional hetero-skedasticity (GARCH) framework for the SP 500 index returns and the Hong Kong stock market returns respectively. Following Guo, we focus on the Student's *t*, NIG and NRIG distributions and the Chinese stock market. Our results indicate the NIG distribution has the best empirical performance in fitting the Chinese stock market index returns.

Literature Review

Although the GARCH model itself could predict unconditional heavy tails, there are still many studies indicating conditional heavy tails after controlling the GARCH effects. For instance, Bollerslev (1987) incorporate the Student's *t* distribution and the GARCH model with the Student's *t* distribution could capture dynamics of a variety of foreign exchange rates and stock price indices returns. Politis (2004) incorporate the truncated standard normal distribution into the ARCH model and demonstrated that the empirical performance of the new type of heavy-tailed distribution on three real datasets. Tavares, Curto and Tavare (2007) model the heavy tails and asymmetric effect on stocks returns volatility into the GARCH framework, and showed the Student's t and the stable Paretian with ($\alpha < 2$) distribution clearly outperform the Gaussian distribution in fitting S&P 500 returns and FTSE returns. Su and Hung (2011) provide a comprehensive analysis of the possible influences of jump dynamics, heavy-tails, and skewness with regard to Value at Risk (VaR) estimates through the assessment of both accuracy and efficiency. Su and Hung consider a range of stock indices across international stock markets during the period of the U.S. Subprime mortgage crisis, and show that the GARCH model with normal, generalized error distribution (GED) and skewed normal distributions provide accurate VaR estimates.

As one of the largest stock market, the Chinese stock market has gained many attentions since its establishment in 1990s. In the following, we will review some of the recent results. Kang, Cheong and Yoon (2010) examined the long memory property in the volatility of Chinese stock markets, and concluded that the volatility of Chinese stock markets exhibits long memory features, and that the assumption of non-normality provides better specifications regarding long memory volatility processes. Sua and Fleisherb (1999) investigated different volatility behaviors between the domestic A-shares and foreign B-shares listed in the Chinese exchanges and suggested some underlying causes of A- and B-share volatility behavior in the Chinese stock markets. Zhang and Li (2008) studied the asymmetric behavior of stock returns and volatilities in the Chinese stock markets and showed that index returns do have asymmetric adjustment behaviors in most of periods and the market tends to overreact to information contained in negative returns and no asymmetry volatility effect was present at the initial stages of the stock market. Xu, et al. (2011) investigated the issue of modelling Chinese stock returns with stable distribution and showed an α-stable distribution is better fitted to Chinese stock return data in the Shanghai Composite Index and the Shenzhen Component Index than the classical Black–Scholes model.

In this paper, we follow the model framework in Guo (2017) and focus the empirical performance of the Student's *t*, NIG and NRIG distributions in fitting the stock market returns in China. The remainder of the paper is organized as follows. In Section 2, we discuss GARCH models and the heavy-tailed distributions. Section 3 discusses the data. The estimation results are in Section 4. Section 5 concludes.

2 The Models

We consider a simple GARCH(1,1) process as:

$$\varepsilon_t = \mu + \sigma_t e_t \tag{2.1}$$

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\ \ 2} + \beta_1 \sigma_{t-1}^{\ \ 2} \tag{2.2}$$

where the three positive numbers α_0, α_1 and β_1 are the parameters of the process and $\alpha_1 + \beta_1 < 1$. The assumption of a constant mean return μ is purely for simplification and reflects that the focus of the paper is on dynamics of return volatility instead of dynamics of returns. The variable e_t is identically and independently distributed (*i.i.d.*). Three types of heavy-tailed distributions are considered: the Student's *t*, the normal inverse Gaussian (NIG) and the normal reciprocal inverse Gaussian (NRIG) distributions. The density function of the standard Student's *t* distribution with ν degrees of freedom is given by:

$$f(e_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^{\ 2}}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \nu > 4 . \tag{2.3}$$

where ψ_{t-1} denotes the σ-field generated by all the available information up through time $t-1$.

The NIG and the NRIG are two special classes of the widely-used generalized hyperbolic distribution. The generalized hyperbolic distribution is specified as in Prause (1999):

$$f(e_t \mid \lambda,\mu,\alpha,\beta,\delta)=\frac{(\sqrt{\alpha^2-\beta^2}/\delta)^{\lambda}K_{\lambda-1/2}(\alpha\sqrt{\delta^2+(e_t-\mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2+(e_t-\mu)^2}/\alpha)^{1/2-\lambda}K_{\lambda}(\delta\sqrt{\alpha^2-\beta^2})}\exp(\beta(e_t-\mu)), \tag{2.4}$$

where $K_{\lambda}(\cdot)$ is the modified Bessel function of the third kind and index $\lambda \in \mathbb{R}$ and: $\delta>0$, $0\le|\beta|<\alpha$. When $\lambda=-\frac{1}{2}$, the Bessel function in the denominator has a closed-form solution, and we have the normalized NIG distribution as:

$$f(\varepsilon_t \mid \psi_{t-1})=\frac{\alpha^2K_1(\frac{\alpha}{\sigma_t}\sqrt{\alpha^2\sigma_t^2+\varepsilon_t^2})}{\pi\sqrt{\alpha^2\sigma_t^2+\varepsilon_t^2}}\exp(\alpha^2). \tag{2.5}$$

When $\lambda=\frac{1}{2}$, we have the normalized NRIG distribution as:

$$f(\varepsilon_t \mid \psi_{t-1})=\frac{\alpha K_0(\sqrt{(\alpha^2-1)^2+\frac{\alpha^2\varepsilon_t^2}{\sigma_t^2}})}{\pi\sigma_t}\exp(\alpha^2-1). \tag{2.6}$$

3 Data and Summary Statistics

The empirical performance of GARCH models with heavy-tailed distribution is explored by using the Chinese stock market returns series. The standardized SHA daily dividend-adjusted close returns are collected from Yahoo Finance for the period from December 19, 1990 to July 19, 2017, covering all the available data in Yahoo Finance. There are in total 6701 observations. Figure 1 exhibits the dynamics of the SHA returns, and the figure exhibits significant volatility clustering.

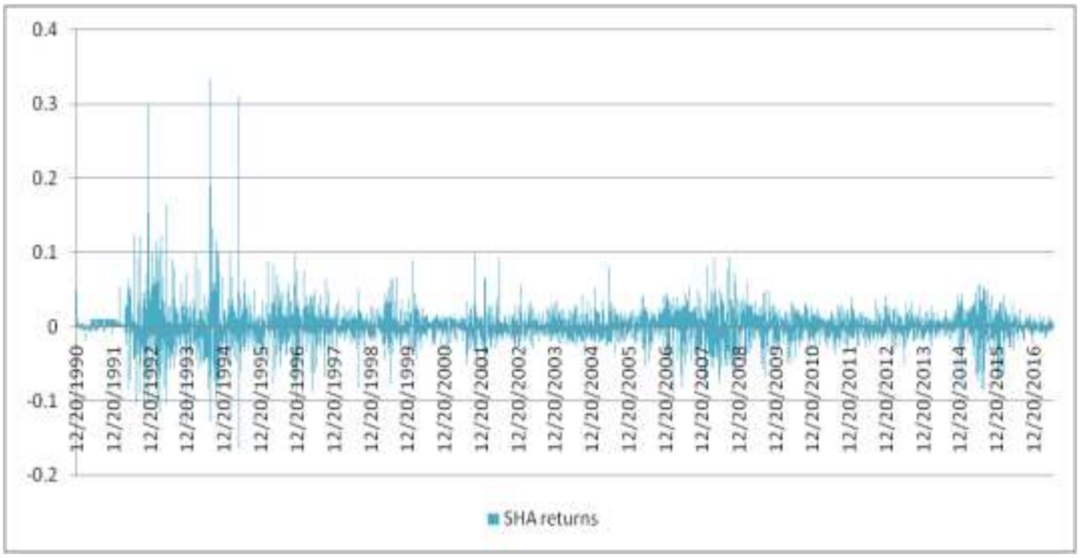

Figure 1: SHA returns

Summary statistics of the data are reported in Table 1. The data illustrate the standard set of well-known stylized facts of asset prices series: non-normality, limited evidence of short-term predictability and strong evidence of predictability in volatility. The Bera–Jarque test conclusively rejects normality of the returns, which confirms the assumption that the model selected should account for the heavy-tail phenomenon. The smallest test statistic is much higher than the 5% critical value of 5.99. The market index is positively skewed and has fat tails. The asymptotic SE of the skewness statistic under the null of normality is $\sqrt{6/T}$, and the SE of the kurtosis statistic is $\sqrt{24/T}$, where T is the number of observations. The series exhibits statistically significant leptokurtosis, suggesting that accounting for heavy-tailedness is more pressing than skewness in modelling asset prices dynamics.

Table 1: Summary statistics. BJ is the Bera-Jarque statistic and is distributed as chi-squared with 2 degrees of freedom, Q(5) is the Ljung-Box Portmanteau statistic, QARCH(5) is the Ljung-Box Portmanteau statistic adjusted for ARCH effects following Diebold (1986) and Q2(5) is the Ljung-Box test for serial correlation in the squared residuals. The three Q statistics are calculated with 5 lags and are distributed as chi-squared with 5 degrees of freedom.

Series	*Obs.*	*Mean*	*Std.*	*Skewness*	*Kurtosis*	*BJ*	*Q(5)*	$Q^{ARCH}(5)$	$Q^2(5)$
DAX	6700	0.06%	2.17%	1.83**	29.20**	219.2**	12.43**	7.83**	89.14**

* and ** denote a skewness, kurtosis, BJ or Q statistically significant at the 5% and 1% level respectively.

The Ljung-Box portmanteau, or Q, statistic with five lags is applied to test for serial correlation in the data, and adjust the Q statistic for ARCH models following Diebold (1986). The evidence of linear dependence in the squared demeaned returns, which is an indication of ARCH effects, is significant for the series.

4 Estimation Results

The GARCH(1,1) model with the Student's *t*, the NIG and the NRIG distributions is estimated by maximizing the following log-likelihood function of equation:

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} \log(f(\varepsilon_t \mid \varepsilon_1, \cdots, \varepsilon_{t-1})) \tag{4.1}$$

Table 2 reports estimation results of the GARCH(1,1) model with the three types of heavy-tailed distribution for all the SHA return series. All the parameters are significantly different from zero. There results show the NIG distribution has better in-sample performance. Since the three distributions has the same number of parameters, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) also indicate the NIG distribution has better empirical performance.

Table 2: Estimation of the GARCH model with heavy-tailed innovations

	alpha0	beta1	1/nu (1/alpha)	log-likelihood
Student's *t*	0.024**	0.935**	0.169**	-11254.3
NIG	0.027**	0.936**	0.824**	-11179.8
NRIG	0.031**	0.924**	0.782**	-11213.5

* and ** denote statistical significance at the 5% and 1% level respectively.

5 Conclusion

Following Guo (2016, 2017), we have investigated the empirical performance of three types of heavy-tailed distributions, the Student's *t*, the normal inverse Gaussian and the normal reciprocal inverse Gaussian, under the GARCH framework and focused on the Chinese stock market index returns. Our results indicate the NIG has the best empirical performance in capture the SHA returns dynamics. Guo (2017) creatively showed that the generalized hyperbolic distribution performs well in risk management of the US stock return series, which is a big finding for stock market practitioners. Thus, it would be interesting to extend the framework further to see how the NIG and the NRIG distributions perform in risk management of the Chinese stock return series, which further benefit the stock market participants in China. Recently, the Shenzhen Stock Exchange (SZSE) plays a more and more important role in the Chinese stock markets, and it would valuable to extend the framework to the SZSE Composite Index returns series. Finally, it will be interesting to consider asymmetric response of conditional volatilities to negative and positive shocks in the GARCH framework as in Zakoian (1994). These are all left for future research.

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